ELEMENTS

OF

MACHINE DESIGN

BY

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PREFACE

THIS little volume is intended primarily as a class-room text book on the subject of elementary machine design. It is the result of the author's experience, extending over more than fifteen years, both in practical design and in teaching of this subject. The pre-requisites on the part of the student are thorough courses in machine drawing and elementary mechanics.

Based on the brief outline of the strength of materials given in Chapter I the author has attempted to develop the equations for the design of the more common machine elements. This has generally been done very concisely and frequently only an outline of the deduction has been given. Empirical formulæ and rule of thumb methods, so much used in elementary texts on this subject, have been avoided as far as possible. There are many factors which in practice affect the design of machine parts that cannot be discussed profitably in the class-room; for instance, cost of construction, capacity of shop machinery, etc. For this reason the teacher must be content if the student acquires the power to analyze the forces and the resultant stresses in machine parts and to apply the proper equations for their design.

The illustrations have been carefully chosen to show typical constructions rather than a great variety which tend to confuse the inexperienced student. The standard text books as well as the American and European technical press have been freely consulted in the preparation of the manuscript.

H. L. NACHMAN.

CHICAGO, March, 1918.

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ERRATA

Page 11. Last figure upside down.
12. Line 14, *for* pin ends *read* flat ends.
12. Line 15, *for* flat ends *read* pin ends.
59. Equation (7) should read $\sqrt[3]{\frac{T_t}{s_s}}$.
62. Prob. 1, *for* tension *read* torsion.
92. Prob. 3, *for* 500,000 *read* 50,000.
116. Fig. 11–4, pitch radius R_1 at large end of gear is omitted.
160. Equation (2) should read $c_t=\frac{V^2}{10}$ (nearly).
169. Equation (22) should read $T_h=R_A(l_1+l_4)-Pl_4$.
171. Equation (28) should read $\frac{bh^2}{6}\overline{\gtrless}\frac{l}{6}\left\{(d+2t)^2-\frac{d^3}{d-2t}\right\}$.
177. Equation (34) should read $s=\frac{M}{z}=\frac{3P}{2}\left(\frac{L-\frac{D}{2}}{bt^2}\right)$.
178. In sketch for prob. 4 crank radius = 18″.
204. Equation (1) should read $t=.02D+\frac{3}{8}''$.
211. Equation (9) *for* P_b *read* P_t.
211. Equation (10) *for* T *read* P_t.

it is gradually increased there will be reached a point where if the load is removed the bar does not resume its original form. This point is called the *elastic limit* of the material. If the load is still further increased rupture of the material will occur sooner or

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CHAPTER I

STRENGTH OF MATERIALS

Elasticity. When an external force acts on a (rigid) body it produces a change of shape in that body. This deformation is called *strain.* When the force is removed the body returns to its original shape. This property which enables a body to resume its initial shape after the application of an external force is called the elasticity of the material.

Load. The external forces acting on a machine part or structure are called *loads.* A *dead load* being one which has a constant value while a *live load* is one which continually changes in value. Thus the load on a column supporting the roof of a building is a dead load, while the forces acting on the connecting rod of a steam engine constitute a live load. If the load be suddenly applied or removed the body carrying such a load is said to be subjected to shock.

Stress. When a body is strained by the application of an external force there are set up internal forces between the particles or molecules which constitute it, which hold the external forces in equilibrium. These internal forces are called *stresses.* The nature and magnitude of the stresses depends on the method of application of the external forces.

Factor of Safety. If a load be applied to a bar of metal and it is gradually increased there will be reached a point where if the load is removed the bar does not resume its original form. This point is called the *elastic limit* of the material. If the load is still further increased rupture of the material will occur sooner or

later. The load at which such failure takes place is called the *ultimate load* and the stress produced the *ultimate stress.* In practice of course no part of a machine should be loaded to anywhere near its ultimate load. The actual load which the part may be permitted to carry is called the *safe load* and the stress produced by this load the *safe stress.* The ratio of ultimate to safe stress is called the *factor of safety* (F) or

$$F=\frac{\text{ultimate stress}}{\text{safe stress}}.$$

In the choice of a proper factor of safety the machine designer must use a good deal of judgment, as its value varies between very wide limits, depending on the material, the kind of load and the purpose of machine. The effect of live loads and shocks is to weaken the material, and experience has shown that the ratio of the factor of safety of any material under dead load, live load and shock should be approximately 1 : 2 : 3; thus, if the factor of safety for any material under dead load is 3, then 6 would be a correct value for a live load and 9 for shock.

Modulus of Elasticity. If a bar be subjected to a constantly increasing load it will be found that within the elastic limit the amount of strain is proportional to the load. That is, if a load of 1000 lbs. be applied to the bar and it is stretched .001 in. a load of 2000 lbs. will stretch it .002 in. The ratio of the stress produced by a load to the strain is called the *modulus of elasticity* and is a constant for any one material, viz.:

$$\text{Modulus of elasticity}=\frac{\text{stress}}{\text{strain}}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

this ratio usually being denoted by the letter E.

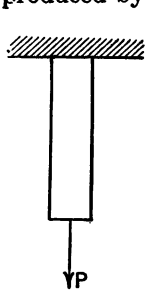


FIG. 1-1.

Tension. There are three kinds of simple stresses, viz.: tension, compression and shear. If a bar be loaded by a force P, acting parallel to the axis of the bar (Fig. 1-1) it produces an elongation, tensile strain, of the bar. If A be the cross-sectional area of the bar and L its length then, since the unit of strain is

always designated per unit (inch) of length, and stress per unit (square inch) of area, we have

$$\text{strain} = \frac{\text{total elongation}}{L},$$

$$\text{stress} = \frac{P}{A}.$$

If we denote the actual or safe stresses for any material by s and ultimate stresses—those at which failure of material occurs—by U or more particularly safe and ultimate *tensile* stresses by s_t and U_t, we have

$$s_t = \frac{P}{A}, \quad \ldots \ldots \ldots \quad (2)$$

or

$$P = As_t, \quad \ldots \ldots \ldots \quad (3)$$

or since $F = \frac{U_t}{s_t}$ this may also be written

$$P = \frac{AU_t}{F}. \quad \ldots \ldots \ldots \quad (3a)$$

In this equation P will be the safe load. If the breaking load be required we have

$$P = AU_t. \quad \ldots \ldots \ldots \quad (4)$$

If strain be denoted by δ we have the equation for modulus of elasticity.

$$E = \frac{\text{stress}}{\text{strain}} = \frac{s}{\delta} = \frac{P}{A\delta}. \quad \ldots \ldots \ldots \quad (5)$$

EXAMPLE 1. A wrought iron tube 2 in. external and $1\frac{1}{2}$ in. internal diameter supports a tensile load of 12,000 lbs. Find stress in pounds per square inch and total elongation of tube if $E = 29{,}000{,}000$ and tube is 8 ft. long, also factor of safety.

Cross-sectional area of tube is

$$A = \frac{\pi}{4}(2^2 - 1.5^2)$$

$$= 1.38 \text{ sq.ins.}$$

Therefore stress is

$$s_t=\frac{P}{A}=\frac{12000}{1.38}=8700 \text{ lb. per sq.in.}$$

Factor of safety is

$$F=\frac{U_t}{s_t}=\frac{45000}{8700}=5.17.$$

From (5) we have

$$\delta=\frac{P}{EA}=\frac{12000}{29000000\times1.38}$$

$$=.0003 \text{ in.}$$

This is the elongation per inch of length and therefore

$$\text{total elongation}=8\times12\times.0003$$

$$=.0288 \text{ in.}$$

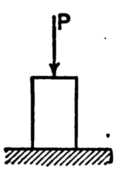


FIG. 1–2.

Compression. If a block be loaded by a force P parallel to its axis in such a way as to shorten it (Fig. 1–2); P constitutes a compressive load and produces compressive stresses and strains in the material. Using the same notation with the addition of U_c for the ultimate compressive stress and s_c for safe or actual compressive stress we have

$$s_c=\frac{P}{A}. \quad \ldots \ldots \quad (6)$$

$$P=As_c, \quad \ldots \ldots \quad (7)$$

and since

$$F=\frac{U_c}{s_c},$$

$$P=\frac{AU_c}{F}. \quad \ldots \ldots \quad (8)$$

EXAMPLE 2. The estimated weight of a building is 840 tons. This weight is to rest on twenty brick piers. If a stress of 150 lbs. per square inch be allowed, what is the size of these piers, assuming them to have a square section?

Load on each pier is

$$P=\frac{840}{20}=42 \text{ tons}$$

$$=84000 \text{ lbs.}$$

Cross-sectional area of pier from (6) is

$$A=\frac{P}{s_c}=\frac{84000}{150}=560 \text{ sq.ins.}$$

Therefore sides are equal to

$$\sqrt{560}=23.6 \text{ ins., say } 24\times 24 \text{ ins.}$$

Shear. If two equal and opposite forces act in the same plane tending to slide the sections of the body on each side of the plane in opposite directions they produce shearing stresses and

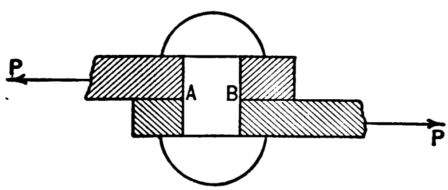


FIG. 1–3.

strains in that plane. Thus in Fig. 1–3 the forces P produce shearing stresses and strains in the section AB of the rivet. Denoting by s_s and U_s the safe and ultimate shearing stresses,

$$s_s=\frac{P}{A}. \qquad (9)$$

$$P=As_s. \qquad (10)$$

$$P=\frac{AU_s}{F}. \qquad (11)$$

TABLE 1

Material.	Modulus of Elasticity. Tension or Compression. E	Modulus of Elasticity. Shear. E_s	Ultimate Stress. Tension. U_t	Ultimate Stress. Compression. U_c	Ultimate Stress. Shear. U_s
Cast iron	14,000,000	6,000,000	18,000	90,000	25,000
Wrought iron	26,000,000	10,000,000	50,000	38,000	40,000
Steel, mild	30,000,000	12,000,000	70,000	80,000	50,000
Steel, tool	32,000,000	....	120,000		
Steel, wire	....	....	120,000		
Copper, cast	12,000,000	....	24,000	65,000	24,000
Copper, hard drawn	15,000,000	....	32,000		
Brass	9,000,000	....	20,000	12,000	
Phosphor-bronze	14,000,000	....	58,000	......	43,000
Manganese-bronze	....	....	60,000	120,000	
Aluminum, cast	9,000,000	....	20,000	12,000	12,000
Aluminum, rolled	10,000,000				
Ash	Average for timber 1,500,000	Average across grain 400,000		6800	
Oak				7000	
Yellow pine				5400	
Red pine				6300	
Spruce				5700	

Compound Stresses. Besides these simple stresses there are three compound stresses, viz.: *bending*, *buckling*, and *twisting* stresses. If a bar be supported at one or both ends, Fig. 1–4, and a load P is applied at right angles to its axis, it will produce *bending* stresses. If a bar which is long in comparison to its other dimensions be supported at one or both ends and loaded by a force P parallel to its axis, it will produce *buckling* stresses. (Fig. 1–5.) A bar held at one point and acted upon by a couple which tends to turn it upon its axis is subjected to *twisting* as shown in Fig. 1–6.

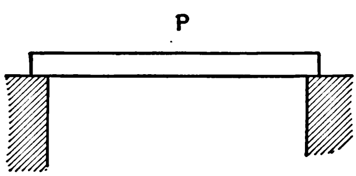


FIG. 1–4.

Bending. If a beam of rectangular section be subjected to bending as in Fig. 1–7 it will take the curved form shown. The material in top of beam will be compressed, that in bottom

stretched, while that in center will be neither stretched nor compressed. This plane along which there is no strain and therefore no stress is called the *neutral plane*. This plane passes through the gravity axes of the sections of beam.

The pressures exerted by the supports upon the beam are called the *reactions*. They can in the simpler cases of loading be readily determined by equating the sum of the moments of all external forces acting, about one of the supports, to zero. Thus in Fig. 1–8, let l be the distance between points of support, called the span of beam, P_1 and P_2 are the loads, R_A and R_B are the reactions at the supports;

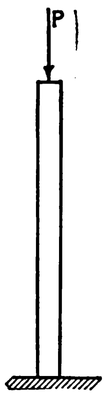


FIG. 1–5.

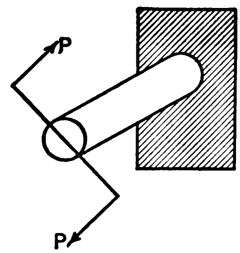


FIG. 1–6.

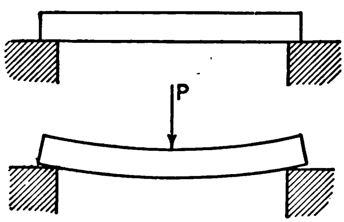


FIG. 1–7.

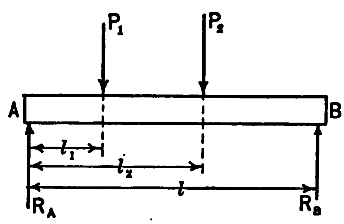


FIG. 1–8.

then taking moments about A and equating their sum to zero we obtain:

$$R_B l - P_2 l_2 - P_1 l_1 = 0$$

and

$$R_B = \frac{P_2 l_2 + P_1 l_1}{l}.$$

The other reaction may be found by taking moments about B or more simply from the fact that the sum of the reaction equals the sum of the loads, thus

$$R_A + R_B = P_1 + P_2,$$

$$\therefore\ R_A = P_1 + P_2 - R_B.$$

Bending Moment. Imagine any section (Fig. 1–9) distant x from one of the supports. If moments of all the external forces on *one* side of this section be taken about the section the sum of these moments is called the *bending moment* at that section and will be designated by M. Thus in Fig. 1–9 the external forces to the left of section at x are R_A, P_1 and P_2, and the bending moment is

$$M = R_A x - P_1(x - l_1) - P_2(x - l_2).$$

The bending moments will thus be different for various sections along the beam. The position at which M is a maximum is called the *dangerous section* because this is the point at which the stress will be a maximum if the beam be one having a uniform

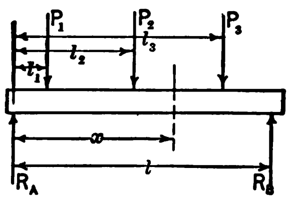


FIG. 1–9.

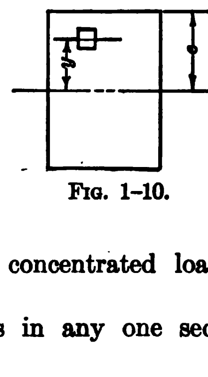


FIG. 1–10.

section throughout its length. With concentrated loads this is always under one of the loads.

The maximum stress which occurs in any one section is given by the equation

$$s = \frac{Me}{I}. \qquad (12)$$

In this equation I is the moment of inertia of the section about its neutral axis and e is the distance from the neutral axis to outside of section. If Fig. 1–10 represent the section of a rectangular beam and the small area a be multiplied by its distance y, from the neutral axis xx, squared, the sum of all the small areas which constitute the section multiplied by the square of their respective distances from the axis xx is called the moment of inertia, I, of the section. The moment of inertia, I, divided by the distance e, is called the section modulus and will be denoted

by the letter z. Table 2 gives moments of inertia of various sections together with their section moduli. From equation (12),

$$M=\frac{sI}{e},$$

$$=sz. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The use of this equation is most easily shown by an example.

EXAMPLE 3. A beam of 10 ft. span carries a load of 4000 lbs. 4 ft. from left support (Fig. 1–11). The beam has a square sec-

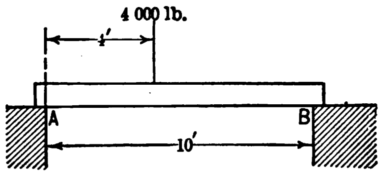


FIG. 1–11.

tion. Find dimensions of sections so that stress will not exceed 1500 lbs. per square inch.

Reaction at A

$$R_A=\tfrac{6}{10}\times 4000=2400 \text{ lbs.}$$

Dangerous section is under load and bending moment at this section is

$$M=2400\times 4\times 12=115200 \text{ in lbs.}$$

$$M=sz.$$

From table 2 $z=\tfrac{1}{6}b^3$,

$$\therefore \quad b=\sqrt[3]{\frac{6\times 115200}{1500}}=7.7 \text{ ins. say 8 ins.}$$

A load acting at one point on a beam is called a concentrated load, while if it is spread out over the entire beam or portions of it, it is called a distributed load. Concentrated loads will be designated by P, and distributed loads by Q. Table 3 gives data necessary for the solution of all beam problems which are likely to be met with in the subject of machine design.

TABLE 2

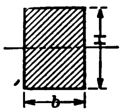
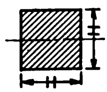
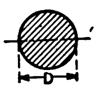
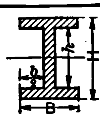
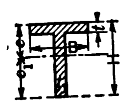
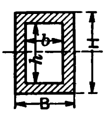
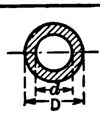
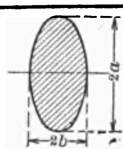
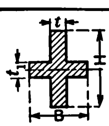
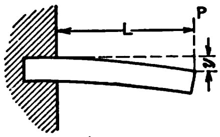
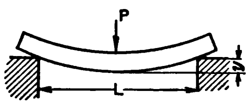
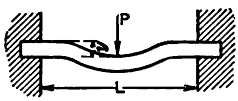
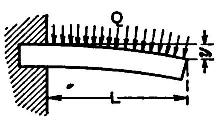
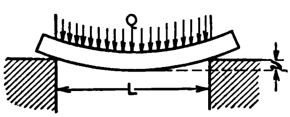
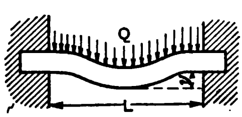
Section.	Moment of Inertia. I	Section Modulus. z
	$\frac{bH^3}{12}$	$\frac{bH^2}{6}$
	$\frac{H^4}{12}$	$\frac{H^3}{6}$
	$\frac{\pi}{64}D^4$	$\frac{\pi}{32}D^3$
	$\frac{BH^3-bh^3}{12}$	$\frac{BH^3-bh^3}{6H}$
	$e=\frac{1}{2}\,\frac{t_1H^2+bt^2}{t_1H+bt}$; $I=\frac{1}{3}(Be^3-bh^3+t_1e_1^3)$	$z=\frac{I}{e}$ or $\frac{I}{e_1}$
	$\frac{BH^3-bh^3}{12}$	$\frac{BH^3-bh^3}{6H}$
	$\frac{\pi}{64}(D^4-d^4)$	$\frac{\pi}{32}(D^3-d^3)$
	$\frac{\pi}{4}a^3b$	$\frac{\pi}{4}a^2b$
	$\frac{tH^3+bt_1^3}{12}$	$\frac{tH^3+bt_1^3}{6H}$

TABLE 3

Beam.	Max. Bending Moment. M	Dangerous Section.	Deflection. Y
	PL	At support	$\frac{PL^3}{3EI}$
	$\frac{PL}{4}$	At middle	$\frac{PL^3}{48EI}$
	$\frac{PL}{8}$	At A and B	$\frac{PL^3}{192EI}$
	$\frac{QL}{2}$	At support	$\frac{QL^3}{8EI}$
	$\frac{QL}{8}$	At middle	$\frac{5QL^3}{384EI}$
	$\frac{QL}{24}$	At A and B	$\frac{QL^3}{384EI}$

Buckling. The commonest example of pieces subjected to buckling stresses are columns. The strength of a column depends largely on how its ends are held. In some cases it is not possible to predetermine whether a piece will fail by buckling or by compression. It is then of course necessary to determine the safe load for both and use the smaller value. Table 4 gives safe loads for various types of columns. These equations are known as Euler's column equations.

Euler's equations are suitable only for very long columns, that is such in which the length is not less than about 45 times the smallest dimension of the cross-section. As the majority of struts and columns used in machine construction are shorter another equation will be found more suitable; this is known as the Rankine equation. For ~~flat~~ ends it is

$$P=\frac{s_cA}{1+k\dfrac{l^2}{\frac{I}{A}}},$$

for ~~pin~~ ends

$$P=\frac{s_cA}{1+4k\dfrac{l^2}{\frac{I}{A}}},$$

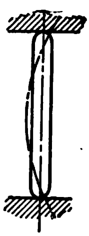
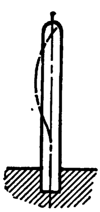
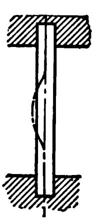
A = area of section of column,
k = a constant determined by experiment,
I = the least moment of inertia of section.

Substituting the proper values of I we have for column of circular section of diameter d.

(Flat ends) $$P=\frac{s_cA}{1+k\dfrac{l^2}{\frac{d^2}{16}}}.$$

(Pin ends) $$P=\frac{s_cA}{1+4k\dfrac{l^2}{\frac{d^2}{16}}}.$$

TABLE 4

Column.		Safe Load P.	
	Held at one end free at the other.	$P=\dfrac{\pi^2 IE}{4Fl^2}$	NOTE.—I = *least* mof ment of inertia o- column section.
	Pin ends guided along original axis.	$P=\dfrac{\pi^2 IE}{Fl^2}$	
	Held at one end, the other end guided.	$P=\dfrac{2\pi^2 IE}{Fl^2}$	
	Both ends held.	$P=\dfrac{4\pi^2 IE}{Fl^2}$	

For rectangular sections, the smallest dimension being denoted by b, this becomes

(Round ends) $$P=\frac{s_cA}{1+k\dfrac{\frac{l^2}{b^2}}{12}}.$$

(Fla ends) $$P=\frac{s_cA}{1+4k\dfrac{\frac{l^2}{b^2}}{12}}.$$

The value of k for

Mild steel is	$\frac{1}{7500}$
Hard steel is	$\frac{1}{5000}$
Wrought iron is	$\frac{1}{9000}$
Cast iron is	$\frac{1}{1600}$
Timber is	$\frac{1}{750}$

Example. What is the safe load on a strut having a rectangular section 2 by 4 ins. and a length of 5 ft? The material is mild steel and the safe stress is 8000 lbs. per square inch. Round ends are to be assumed.

As $\dfrac{l}{b}=\dfrac{60}{2}=30$ the Rankine equation is to be used. Then

$$P=\frac{s_cA}{1+k\dfrac{\frac{l^2}{b^2}}{12}}=\frac{8000\times(2\times4)}{1+\dfrac{1}{7500}\dfrac{\frac{(60)^2}{(2)^2}}{12}}$$

$$=26200 \text{ lbs.}$$

Torsion. The principal stress induced by torsion is a shearing stress and will therefore be denoted by s_s. The force which turns or tends to turn the shaft, multiplied by its perpendicular distance to center of shaft, Fig. 1–12, is called the twisting moment or torque and will be denoted by T. Thus

$$T=PR.$$

The stress induced in a rod by a torque T is given by the equation

$$s_s = \frac{Te}{I_p}, \qquad (14)$$

and therefore

$$T = \frac{s_s I_p}{e}$$

$$= s_s z_p. \qquad (15)$$

In this equation I_p is the polar moment of inertia and z_p is the polar section modulus. Table 5 gives values of I_p and z_p for the usual sections.

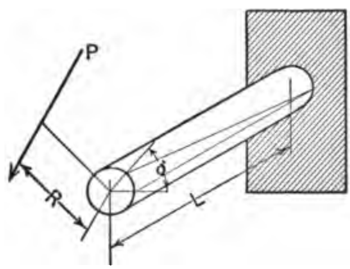


Fig. 1–12.

The angle δ through which a rod subjected to torsion is turned is given by the equation

$$\delta = \frac{180}{\pi} \frac{T}{I_p} \frac{L}{E_s}$$

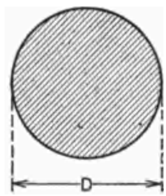
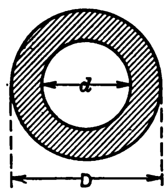
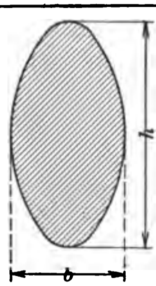
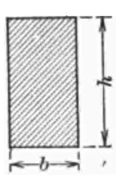
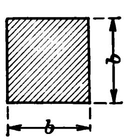
$$= 57.2 \frac{TL}{I_p E_s}. \qquad (16)$$

Equations (14), (15), and (16) are true strictly only for circular sections but may be used with sufficient accuracy for other sections.

Combined Stresses. A machine part is frequently subjected to both bending and torsion. In this case it is usual to figure an ideal twisting moment which if applied would give a stress equivalent to the actual stress due to bending and torsion. This ideal twisting moment (T_i) is then used in equation (15) to find the stress. The value of it is calculated from the equation

$$T_i = M + \sqrt{T^2 + M^2}.$$

TABLE 5

Section.	Polar Moment of Inertia. I_p	Polar Section Modulus. Z_p
	$\frac{\pi}{32}D^4$	$\frac{\pi}{16}D^3$
	$\frac{\pi}{32}(D^4-d^4)$	$\frac{\pi}{16}\frac{(D^4-d^4)}{D}$
	$\frac{\pi}{32}b^3h^3$	$\frac{\pi}{16}b^2h$
	$\frac{1}{12}bh(b^2+h^2)$	$\frac{2}{9}b^2h$
	$\frac{1}{6}b^4$	$\frac{2}{9}b^3$

There are other cases of combined stresses but they will be discussed whenever they occur in the text.

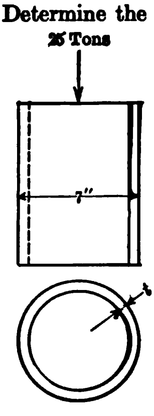
PROBLEMS

1. How much will a steel tube 1¼ in. outside diameter and 1 in. inside diameter stretch when carrying a load of 7000 lbs. The tube is 8′ 6″ in. long. What is the factor of safety if the ultimate tensile resistance is 50,000 lbs. per square inch? $E = 30{,}000{,}000$.

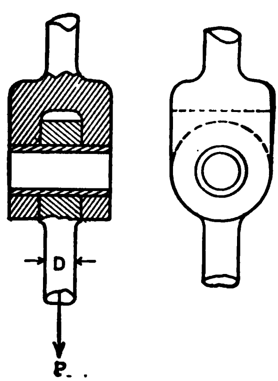
2. A wrought iron bar 1 in. in diameter and 10 ft. long is .015 in. longer when loaded than when under no stress. $E = 29{,}000{,}000$. Determine (*a*) the load on the bar, (*b*) the stress per square inch, and (*c*) the factor of safety if the ultimate stress is 40,000 lbs. per square inch.

3. Sketch shows a hollow cylindrical ring supporting a load of 25 tons. Assuming it to be of cast iron having an ultimate compressive strength of 80,000 lbs. per square inch, what should thickness be if a factor of safety of 20 be used?

4. Find diameter D of tension rod to sustain a load $P = 40{,}000$ lbs., so that the stress in rod will not exceed 10,000 lbs. per square inch. The pin joining the two rods is 2½ in. in diameter, and has a 1½-in. hole through it. Determine the shearing stress in pin.



CHAP. I. Prob. 3.



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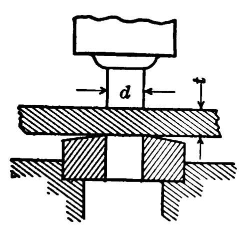


CHAP. I. Prob. 5.

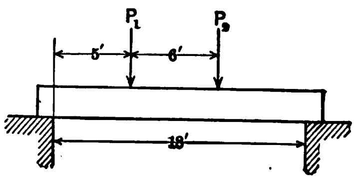
5. Sketch represents a punch and die. If $d = 1$ in., $t = \frac{3}{4}$ in. and shearing strength of plate to be punched is 40,000 lbs. per square inch what force P is required to punch the hole?

6. A beam of 20-ft. span is loaded at four points equidistant from each other and from the two end supports, with four equal loads of 2 tons each. Find (*a*) the two reactions at the supports, (*b*) the bending moments under each load.

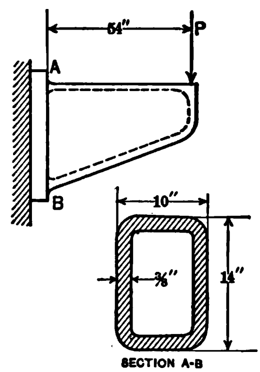
7. Beam is loaded as shown by two concentrated loads, $P_1 = 2500$ lbs. and $P_2 = 1500$ lbs. Find both reactions and the bending moments under

each load. The beam has a square section: find its dimensions if the safe stress is 1000 lbs. per square inch.

8. A cantilever beam is 10-ft. span. It is loaded with a uniformly distributed load of 350 lbs. per running foot and a concentrated load of 4000 lbs., 8 ft. from support. Find maximum bending moment; find dimensions of section if this is a rectangle the depth of which is three times its width. The safe stress is 1200 lbs. per square inch.



Chap. I. Prob. 7.



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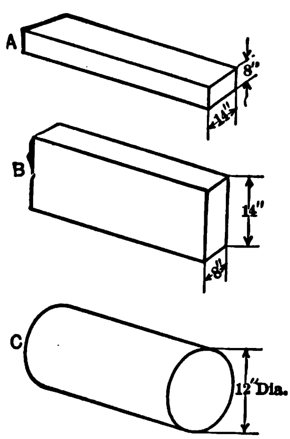
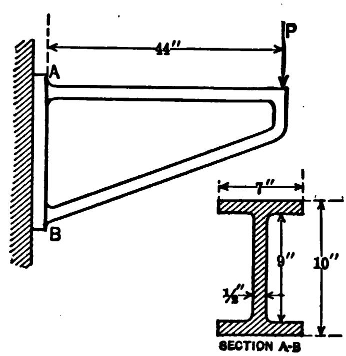


Chap. I. Prob. 9.



Chap. I. Prob. 11.

9. The three cantilevers A, B, and C are of equal cross-sectional area. Assuming same material find ratios of their safe loads.

10. The bracket carries a load $P = 10{,}000$ lbs. The section is as shown. Determine the stress in this section.

11. This bracket has section at support as shown. Determine the load which it can carry if the allowable stress is 6000 lbs. per square inch.

CHAPTER II

SCREW FASTENINGS

Where it is necessary to fasten two or more parts together in such a way that they may be readily separated some form of the screw or bolt is most commonly used. The thread is formed by cutting or rolling a helical groove into the blank, the form of the cross-section of this groove being in general either triangular or rectangular with the corners sharp or rounded. The triangular thread, which is the stronger, is used for screws and bolts the purpose of which is to fasten machine parts together. The rectangular thread, producing less friction, is used where the object of the thread is the transmission of power or motion.

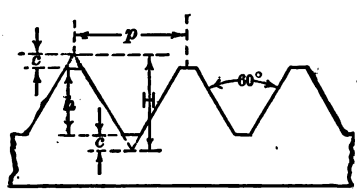


FIG. 2–1.

U. S. Standard Thread. Fig. 2–1 shows the form of the U. S. Standard or Sellers thread. This is an equilateral triangle with top and bottom cut off. The pitch, p, is the distance from one turn of the thread to the next turn. The amount cut off at top and bottom is $c=\frac{H}{8}$ so that $h=\frac{3}{4}H=.65p$. If D is the outside or nominal diameter then diameter at bottom of thread is $d=D-1.3p$. The number of threads per inch is $n=\frac{1}{p}$. Table 1 below gives standard dimensions of bolts as used in this country.

The Whitworth Thread. This thread is used universally in Great Britain and very largely on the Continent. The sides of

TABLE 1

Screw-threads

Diameter of Screw.	Number of Threads per Inch.	Diameter at Bottom of Threads.	Area at Bottom of Threads in Square Inches.	Area of Bolt Body in Square Inches.
1/4	20	.185	.027	.049
5/16	18	.240	.045	.077
3/8	16	.294	.068	.110
7/16	14	.344	.093	.150
1/2	13	.400	.126	.196
9/16	12	.454	.162	.249
5/8	11	.507	.202	.307
3/4	10	.620	.302	.442
7/8	9	.731	.420	.601
1	8	.837	.550	.785
1 1/8	7	.940	.694	.994
1 1/4	7	1.065	.893	1.227
1 3/8	6	1.160	1.057	1.485
1 1/2	6	1.284	1.295	1.767
1 5/8	5 1/2	1.389	1.515	2.074
1 3/4	5	1.491	1.746	2.405
1 7/8	5	1.616	2.051	2.761
2	4 1/2	1.712	2.302	3.142
2 1/4	4 1/2	1.962	3.023	3.976
2 1/2	4	2.176	3.719	4.909
2 3/4	4	2.426	4.620	5.940
3	3 1/2	2.629	5.428	7.069
3 1/4	3 1/2	2.879	6.510	8.296
3 1/2	3 1/4	3.100	7.548	9.621
3 3/4	3	3.317	8.641	11.045
4	3	3.567	9.963	12.566
4 1/4	2 7/8	3.798	11.329	14.186
4 1/2	2 3/4	4.028	12.753	15.904
4 3/4	2 5/8	4.256	14.226	17.721
5	2 1/2	4.480	15.763	19.635
5 1/4	2 1/2	4.730	17.572	21.648
5 1/2	2 3/8	4.953	19.267	23.758
5 3/4	2 3/8	5.203	21.262	25.967
6	2 1/4	5.423	23.098	28.274

the thread make an angle of 55° with each other. The top and bottom are rounded off and an amount $c = \frac{1}{6}H$ is cut off from the primitive triangle. For this thread $d = D - 1.28p$.

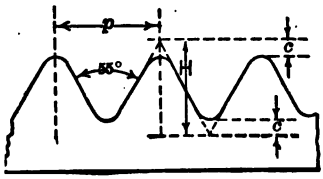


FIG. 2–2.

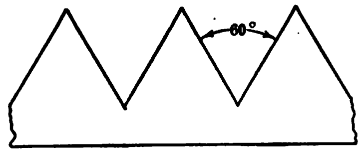


FIG. 2–3.

Other Forms of Threads. The sharp *V* thread is shown in Fig. 2–3. The sides make an angle of 60° with each other. The thread is used both in America and Europe. Since the section

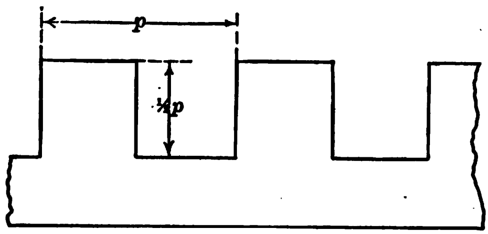


FIG. 2–4.

is an equilateral triangle $d = D - 1.732p$. On account of greater depth and sharp corner at bottom a bolt with this thread is much weakened. Fig. 2–4 shows the square thread used generally

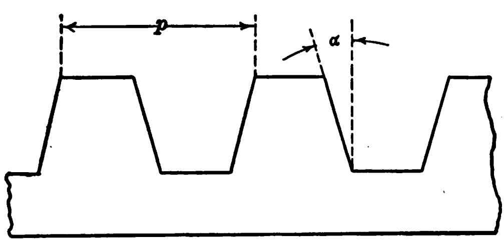


FIG. 2–5.

where the transmission of motion is the purpose of the screw is, for instance, the lead screw of a lathe. With the proportions

shown $d = D - p$. A modification of this thread known as the "acme thread" is shown in Fig. 2-5. The angle $\alpha = 14\frac{1}{2}°$.

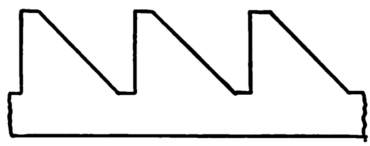


Fig. 2-6.

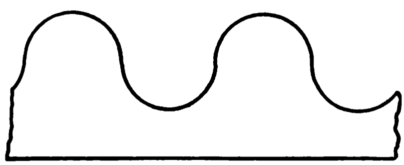


Fig. 2-7.

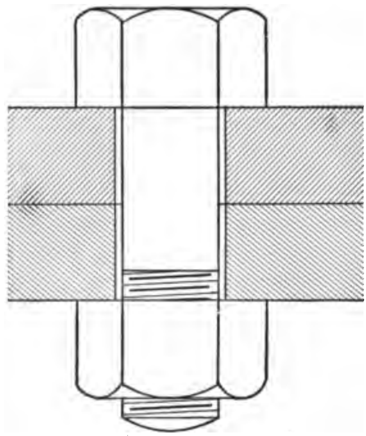


Fig. 2-8.

The "buttress thread," Fig. 2-6, is used where the pressure always acts in one direction, against the perpendicular side, as in the breech mechanism of modern ordnance.

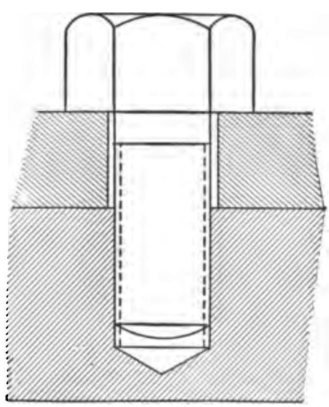


Fig. 2-9.

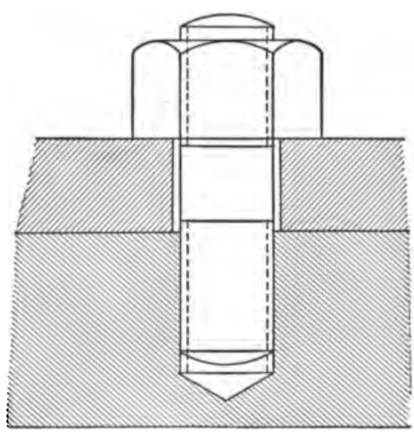


Fig. 2-10.

The "knuckle thread," Fig. 2-7, formed by rounding the top and bottom of the square thread is especially adapted for rough usage as it is not liable to be easily injured

Types of Bolts. Bolts may be divided into three general classes: (*a*) through bolts, (*b*) tap bolts or cap screws and (*c*) stud bolts. In fastening two parts together by a through bolt

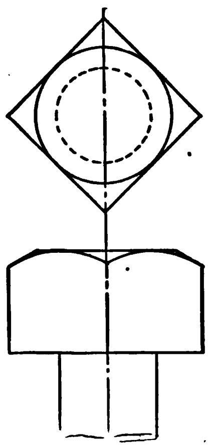


FIG. 2-11.

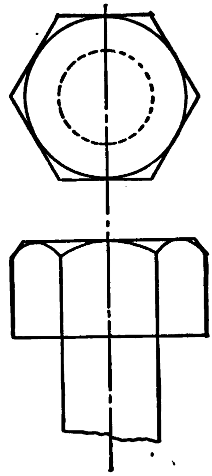


FIG. 2-12.

(Fig. 2-8) the holes are "drilled" an easy fit for the bolt. With tap bolts one hole is drilled and the other tapped or threaded as shown in Fig. 2-9. The stud bolt is threaded at both ends as

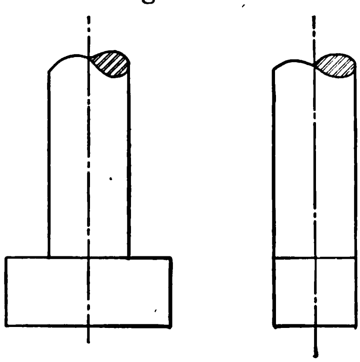


FIG. 2-13.

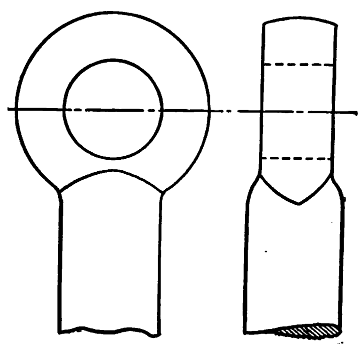


FIG. 2-14.

shown in Fig. 2-10, one hole being drilled and the other tapped. A nut is used to hold the parts together. These bolts are made in sizes from $\frac{1}{4}$ in. up, as shown in Table 1.

Forms of Boltheads. Bolts are made with various forms of heads depending upon the use to which they are to be put. The commonest forms being the square and hexagon heads shown in Figs. 2–11 and 2–12. A few of the other forms frequently met with are shown in Figs. 2–13 to 2–16. Fig. 2–13 is the Tee head, Fig. 2–14 is an eye bolt, Fig. 2–15 is a hook bolt and Fig. 2–16 is the round-head bolt.

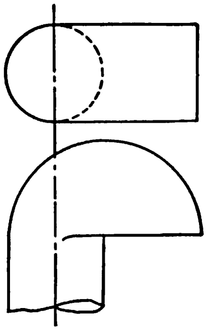


FIG. 2–15.

Machine Screws. The term, machine screw, is applied to a variety of small screws, generally with screw-driver heads, ranging in diameter from $\frac{1}{16}$ to $\frac{1}{2}$ in. No standard shave been adopted for these up to the present time. The usual forms of heads are shown in Figs. 1–17 to 1–19.

Set screws are used to prevent relative rotation of two parts as a pulley and shaft, the holding power being due to friction produced by pressure on end of screw. In Fig. 2–20 is shown the usual form of these; *a* being the flat end, *b* the cone, and *c* the cup end screw.

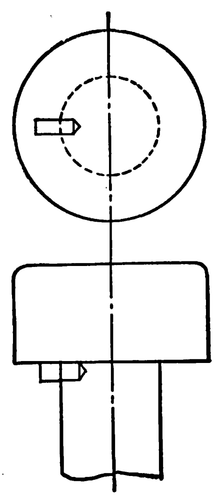


FIG. 2–16.

Locking Device. When a bolt is used where it is subjected to vibration it is necessary to lock the nut to prevent it from loosening. The commonest form of locking device is two nuts, the lower one, which is the lock or check nut; should be tightened against the upper nut (Fig. 2–21). The thickness of check nut is generally about half that of the regular nut. Other methods of locking nuts are shown in Figs. 2–22 to 2–24. In Fig. 2–23 the nut has a cylindrical extension which extends into the part to be held by bolt, a set screw grips this extension. In Fig. 2–24 a washer is placed under the nut, a portion of this washer is bent up to fit against one face, while another portion is bent down into an opening thus preventing washer and nut fom turning.

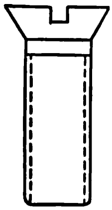


FIG. 2-17.

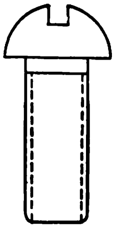


FIG. 2-18.

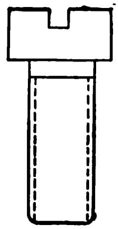


FIG. 2-19.

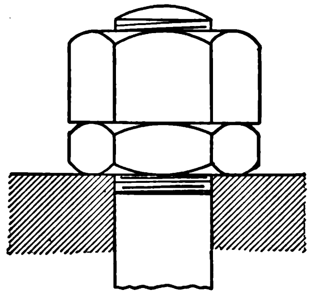


FIG. 2-21.

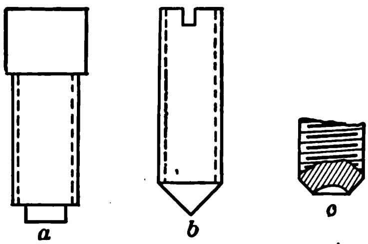


FIG. 2-20.

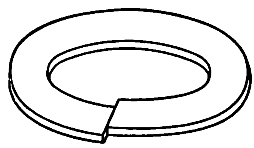


FIG. 2-22.

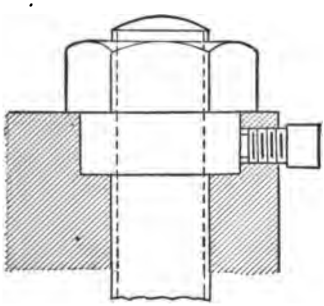


FIG. 2-23.

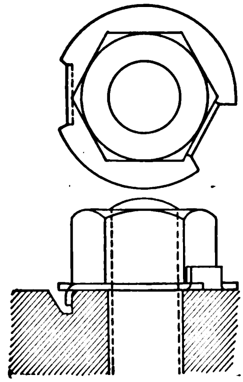


FIG. 2-24.

Pipe Thread. The standard form of pipe thread is the Briggs thread shown in Fig. 2-25. The number of threads per inch are

$\frac{1}{8}$-in. pipe	27	threads	per inch.
$\frac{1}{4}$ and $\frac{3}{8}$-in. pipe	18	"	"
$\frac{1}{2}$ and $\frac{3}{4}$-in. pipe	14	"	"
1 to 2-in. pipe	$11\frac{1}{2}$	"	"
$2\frac{1}{2}$ ins. and over	8	"	"

Multiple Threaded Bolts. A bolt may have two or more threads cut upon it. The distance between adjacent turns is again the pitch, p, while the distance which one thread advances in a complete revolution is usually called the lead, L. Figs. 2–26 and 2–27 illustrate a double and triple threaded bolt. These bolts are used for power transmission purposes as they give a better efficiency than single threaded bolts.

The Strength of Bolts. For purposes of design bolts may be divided into three classes:

1. Those in which the stress is due to the load only.
2. Those which are under an initial stress due to tightening.
3. Those which are used to transmit power or motion.

Fig. 1–28 shows a bolt in which the stress is that due to the load only. If d=root diameter, P is the tensile load and s_t is the safe tensile stress, then

$$P = As_t$$

$$= \frac{\pi}{4} d^2 s_t,$$

and

$$\therefore \quad d = \sqrt{\frac{4P}{\pi s_t}} = 1.13\sqrt{\frac{P}{s_t}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The outside diameter D for the U. S. standard thread may then be obtained from Table 1.

The second case is illustrated by Fig. 2–29 which shows one of the stud bolts holding the cylinder head of a steam engine. Here it is necessary to screw down the nut sufficiently to make a steam tight joint. The amount of this initial tightening depends on the relative elasticities of the material of bolts, flanges and packing.

Let P = pressure to be sustained by each bolt,
k = a coefficient depending on kind of packing,
s_t = safe tensile stress in bolt,
A = root area of bolt,
d = root diameter bolt.

Then

$$A = \frac{P}{S_t} k,$$

$$d = 1.13\sqrt{\frac{P}{S_t}k}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The following values of k may be used: For a ground joint or metallic packing such as a copper ring $k=1\frac{3}{8}$, for soft packing such as paper, asbestos or rubber $k=1\frac{5}{8}$.

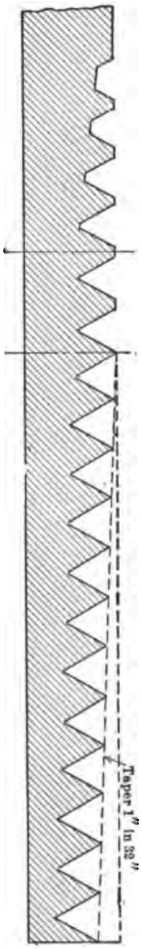


FIG. 2-25.

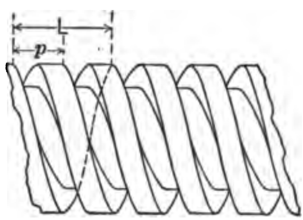


FIG. 2-26.

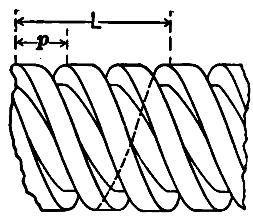


FIG. 2-27.

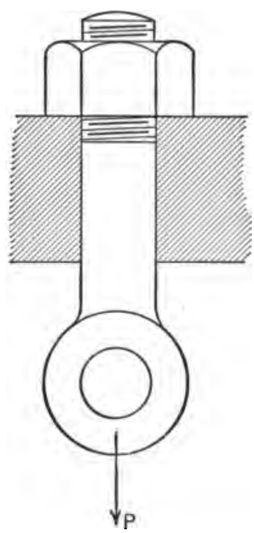


FIG. 2-28.

EXAMPLE. Determine the diameter of bolts necessary to hold the cylinder head of an engine having a cylinder diameter of 10 ins. and using steam at 150 lbs. per square inch. We will assume

that 8 bolts are used and that the gasket is of rubber. The safe stress for these bolts should be assumed low from 4000 to 6000 lbs. per square inch; as this is a case of repeated or continually varying loads.

Total pressure on cylinder head is

$$W=\frac{\pi}{4}\times 100\times 150=11{,}800 \text{ lbs.}$$

Pressure on each bolt is

$$P=\frac{W}{8}=1475 \text{ lbs.}$$

Then assuming $s_t=6000$ we have

$$d=1.13\sqrt{\frac{P}{S_t}}k=1.13\sqrt{\frac{1475}{6000}}\times 1\tfrac{5}{8}$$
$$=.715 \text{ in.}$$

This requires a bolt $\frac{7}{8}$ in. outside diameter.

Transmission Screws are used for the transmission of power in hoisting machines, screw presses, machine tools, such as planers and slotters, and many others. The power may be applied either to the nut or the screw. The stresses induced in screw are tension, or compression, combined with torsion. In Fig. 2–30 let

r_1 = outside radius of thread,
r_2 = inside radius of thread,
r = mean radius of thread,
p = pitch of thread,
μ = coefficient of friction between nut and thread,
μ' = coefficient of friction between collar and support,
R' = mean radius of collar,
α = angle of thread $=\tan^{-1}\frac{p}{2\pi r}$,
ϕ = friction angle $=\tan^{-1}\mu$,
P = force applied at end of lever,
N = normal pressure between nut and screw thread,
W = load on thread.

Since the work done by force applied at end of lever must be equal to the work required to raise the load W, and to overcome friction of nut and of collar, we have for one turn of the lever,

$$2\pi RP = Wp + 2\pi r\mu N + \mu' W 2\pi R',$$

$$N = W \cos \alpha,$$

$$2\pi RP = W[p + 2\pi(\mu r \cos \alpha + \mu' R')].$$

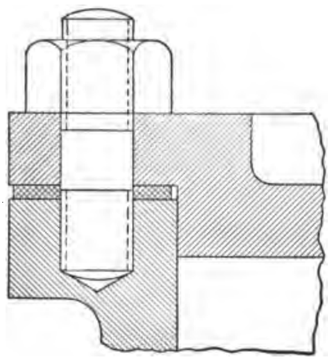


FIG. 2-29.

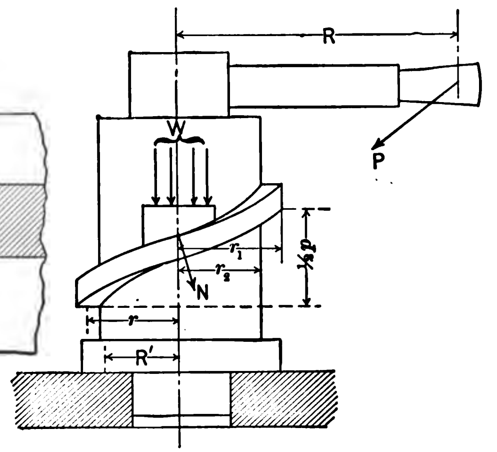


FIG. 2-30.

The efficiency of the screw is

$$e = \frac{\text{useful work done}}{\text{total energy applied}} = \frac{Wp}{2\pi RP}$$

$$= \frac{p}{p + 2\pi(\mu r \cos \alpha + \mu' R')}. \quad \ldots \ldots \quad (3)$$

If $\mu = \mu'$ and $r = R'$ this reduces to

$$e = \frac{\tan \alpha}{\tan \alpha + \tan \phi(\cos \alpha + 1)}. \quad \ldots \ldots \quad (4)$$

The efficiency therefore increases as α increases, and it is for this reason that multiple threaded screws are used for power transmission. If $\tan \alpha$ is less than μ the screw is self-locking, that is, no matter how great the load W may be, it cannot cause rotation of the screw and consequent running down of nut. This is an important consideration in the design of transmission screws and worm gears for hoisting machinery. If $\tan \alpha$ is greater than the

above value a force or load applied to nut may cause rotation of screw. A familiar example of this is the spiral screwdriver and drill.

In calculating the diameter of a transmission screw it is sufficient to make the root diameter safe to resist the direct tension or compression as the increase in strength due to thread will take care of the torsion stresses.

EXAMPLE. Design the screw for a hoisting jack to raise a load of 10 tons. Assume a safe stress of 8000 lbs. per square inch, then the root diameter is

$$d=1.13\sqrt{\frac{P}{s_c}}=1.13\sqrt{\frac{20000}{8000}}$$
$$=1.79 \text{ in.}$$

If a pitch of $\frac{1}{2}$ in. be assumed then the outside diameter of the square threaded screw is

$$d+p=1.79+\tfrac{1}{2}=2.39 \text{ ins., say } 2\tfrac{1}{2} \text{ ins.}$$

The coefficient of friction may be assumed at .10 then

$$\phi=\tan^{-1}.10=5^\circ\ 40',$$

$$\alpha=\tan^{-1}\frac{.5}{2\pi 1.25}=4^\circ.$$

The efficiency of the screw therefore is

$$e=\frac{\tan 4^\circ}{\tan 4^\circ+\tan 5^\circ-40'(\cos 4^\circ+1)}$$

$$=\frac{.07}{.07+.10(.998+1)}=26 \text{ per cent.}$$

It should be noted that, the screw is subjected to a compressive load, it may have to be designed as a column if it be long in comparison to its diameter. At any rate it will be well to check the safe load according to the column equation of Chapter I.

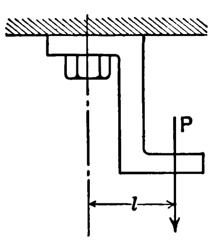


FIG. 2-31.

If the load on a bolt is not applied centrally with the bolt there will be produced a bending stress besides the direct stress. Thus in Fig. 2-31 the bolt is sub-

jected to a combined direct tension P and a bending moment equal to Pl.

PROBLEMS

1. In a screw press a force of 25 tons is transmitted to the platen by means of two square threaded screws of 6 threads to the inch. Determine outside diameter of these screws if a tensile stress of 8000 lbs. be allowed.

2. The cylinder head of a steam engine is held by 10 stud bolts. The diameter of cylinders is 12 ins. and the steam pressure is 125 lbs. per square inch. Find root diameter of bolts if safe stress is 4000 lbs. per square inch.

3. In a turnbuckle the threaded ends are $1\frac{1}{2}$ ins. outside diameter, 4 threads per inch. The material is soft steel having a tensile strength of 40,000 lbs. per square inch. The turnbuckle was tightened until rupture of one of the ends occurred. What was the load on the rods?

4. A jack screw is to be designed to raise a maximum load of 30 tons. Determine its root diameter if a factor of safety of 4 be employed and the material is mild steel.

5. In a hydraulic press the upper platen is held by four bolts. The pressure is exerted by a plunger 15 ins. diameter with a maximum hydraulic pressure of 400 lbs. per square inch. Determine size of bolts if the safe stress is 8000 lbs. per square inch.

6. What is the efficiency of a square threaded screw 3 ins. diameter. It is triple threaded, the lead being 1 in. Assume coefficient of friction is .12.

CHAPTER III

RIVETED JOINTS

Types of Rivets and Joints. When two parts are to be permanently fastened together the riveted joint is commonly used. The rivet consists of a head and a cylindrical shank slightly tapered at the end (Fig. 3–1). It is heated to a bright red heat

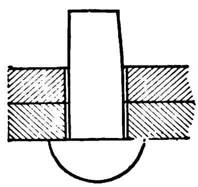


FIG. 3–1.

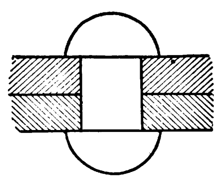


FIG. 3–2.

and another head called "point" is formed, either by pressure or by hammering, as shown in Fig. 3–2. Riveted joints are used chiefly in structural steel work and for connecting the plates of vessels under pressure such as boilers and tanks. In structural work strength of joint is the chief consideration while in boiler

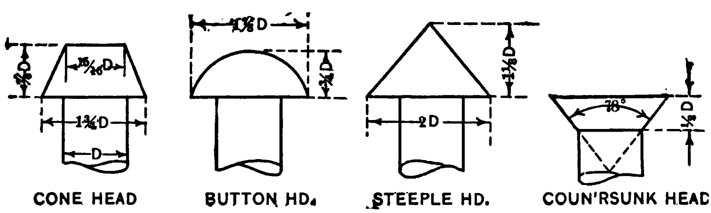


FIG. 3–3.

joints and similar work the tightness of joint to prevent leakage is of equal importance. Fig. 3–3 shows the various forms of rivet heads which are commonly used.

Boiler joints are of two general types called lap joints and butt joints. In the lap joints the plates to be riveted together

overlap as shown in Fig. 3-4. Depending on the number of rows of rivets the joints is single, double, triple or quadruple riveted. In the butt joint shown in Fig. 3-5 the ends of the plates butt

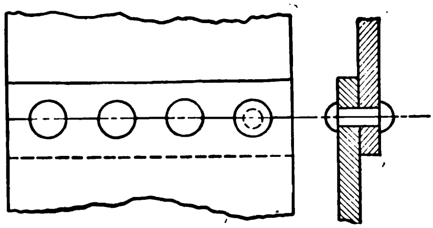


FIG. 3-4.

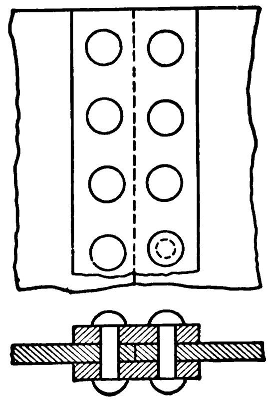


FIG. 3-5.

against each other and a strip of metal, called a butt strap or welt, is placed on one or both sides of the plates. A single riveted butt joint has one row of rivets on each side of joint, a double riveted butt joint has two rows, and so on. The butt joint is

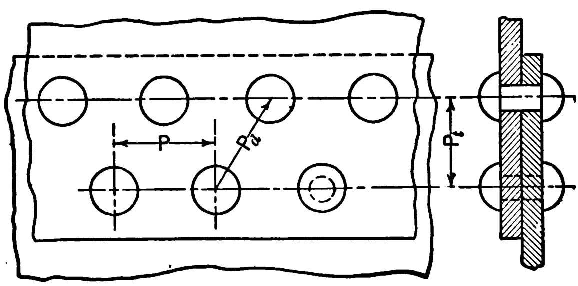


FIG. 3-6.

considered the safest and is generally used in high-pressure boilers. Figs. 3-4 to 3-10 show some of the common forms of boiler joints.

The material used for rivets, plates and structural shapes is usually open-hearth or Bessemer steel, wrought iron being but

rarely used in present day practice. For boiler work open-hearth steel is used almost exclusively, there being three grades, called flange or boiler steel, fire-box steel and extra soft steel. Their tensile and shearing strengths range as follows:

	Tensile Strength.	Shearing Strength.
Flange steel	55,000–65,000	48,000
Fire-box steel	52,000–62,000	48,000
Extra soft steel	45,000–55,000	45,000

Rivets are made of bars of extra soft steel.

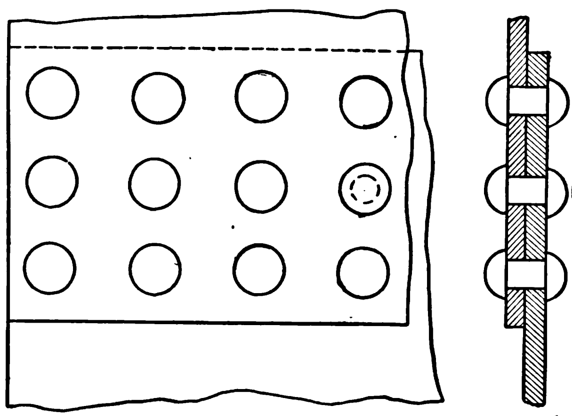


FIG. 3–7.

Calking. Joints are made fluid tight by calking as shown in Fig. 3–11. For this purpose the edges of the plates to be calked are planed off at an angle of about 80°. These edges are then burred down by means of the calking chisel. This work is done either by hand or pneumatic hammer. Skill and care are required to prevent injury of the plate.

Strength of Riveted Joints. The distance between adjacent rivets in the same row is called the pitch of the rivets and will be indicated by p. If the pitch of the rivets in the different rows is not the same, the maximum pitch will be understood by p, as in Fig. 3–9. When the rivets in the various rows are opposite each other it is called chain riveting (Fig. 3–7). If

they are staggered it is called zigzag riveting (Fig. 3–6). The distance between center lines of adjacent rows of rivets is the

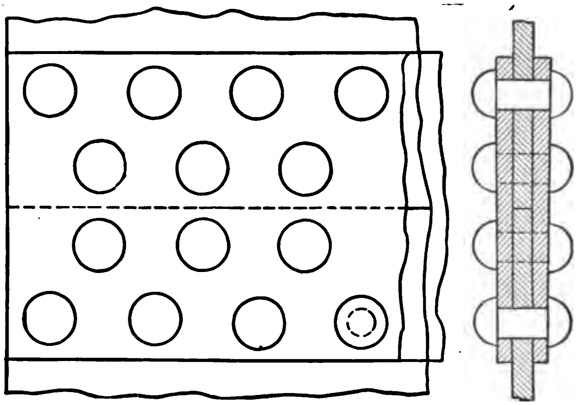


Fig. 3–8.

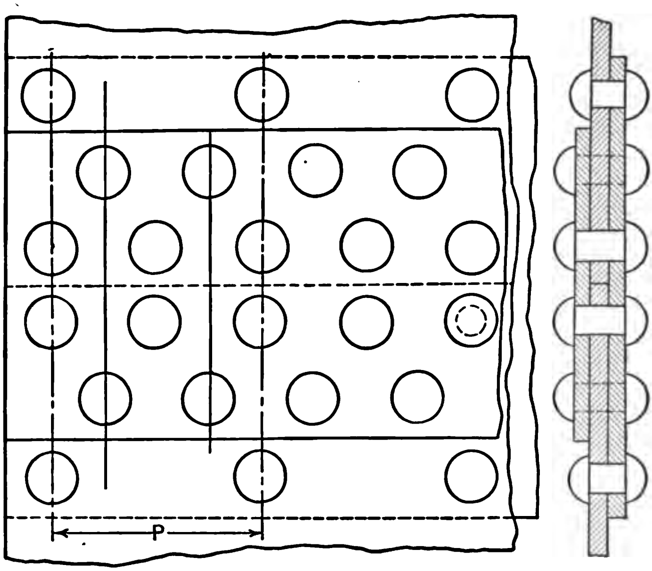


Fig. 3–9.

transverse pitch and will be indicated by p_t, while the diagonal pitch, p_d, is the distance from the center of one rivet to that of the nearest to it, diagonally, in the next row (Fig. 3–6).

Failure of a joint may occur in any one of the following ways:

1. Tearing of plate between rivets as in Fig. 3–12.
2. Shearing of rivets as in Fig. 3–13.
3. Crushing of plate or rivet as in Fig. 3–14.
4. Tearing of plate in front of rivet as in Fig. 3–15.
5. Shearing of plate in front of rivet as in Fig. 3–16.

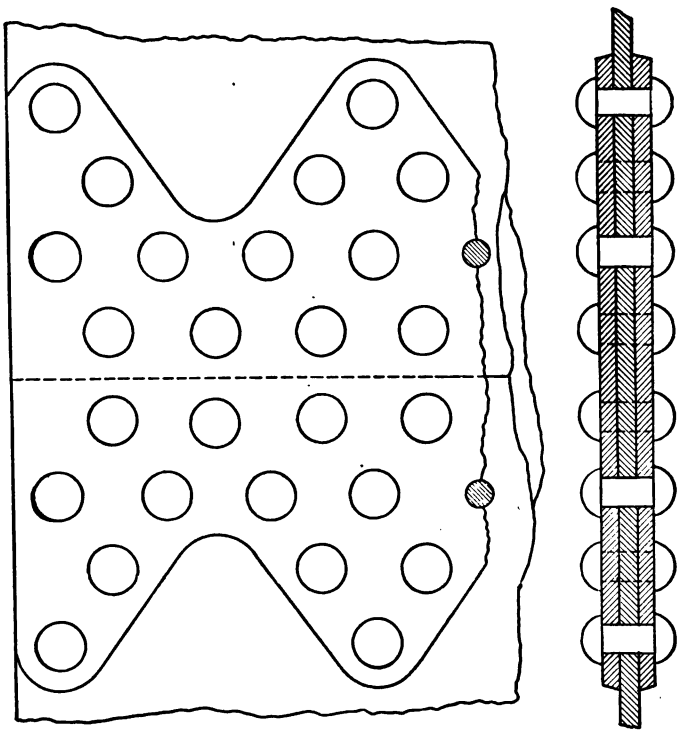


Fig. 3–10.

In the more complex types of joints failure may occur by a combination of two or more of the above causes.

The ratio of the strength of weakest element of the joint to that of the solid plate is called the efficiency of the joint. It is usually expressed in per cent. If the centers of rivets be placed from one and one-half to twice the rivet diameter from the edge of plate, failure by methods 4 and 5 will not occur and therefore

consideration of these methods of failure will be omitted from the following discussion.

Let d = diameter of rivet in inches,
t = thickness of plate in inches,
U_t = tensile resistance of plate,
U_c = crushing resistance of plate or rivet,
U_s = shearing resistance of rivet in single shear.

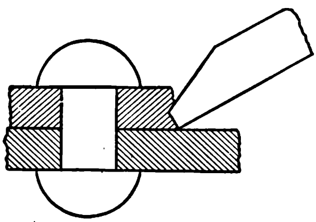


FIG. 3-11.

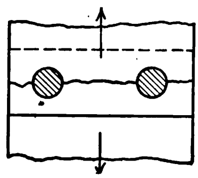


FIG. 3-12.

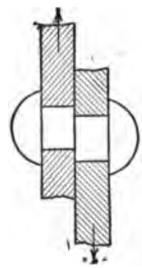


FIG. 3-13.

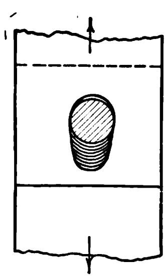


FIG. 3-14.

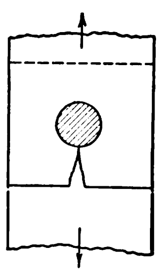


FIG. 3-15.

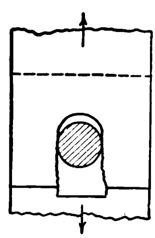


FIG. 3-16.

The shearing resistance of a rivet in double shear will be taken at 1.8 the shearing resistance of the same rivet in single shear. Since portions of the joint of a length equal to the pitch are identical it is only necessary to consider such a part of the joint and it will be called a *unit strip*.

Let R = tensile resistance of a unit strip of the solid plate,
R_t = tensile resistance of a unit strip of the joint,
R_s = shearing resistance of a unit strip of the joint,
R_c = crushing resistance of a unit strip of the joint,

E_t = tensile efficiency of the joint $= \frac{R_t}{R} \times 100$ per cent,

$$E_s = \text{shearing efficiency of the joint} = \frac{R_s}{R} \times 100 \text{ per cent},$$

$$E_c = \text{crushing efficiency of the joint} = \frac{R_c}{R} \times 100 \text{ per cent}.$$

Single Riveted Lap Joint (Fig. 3–4). In this joint there is one rivet in the unit strip. The cross-sectional area of a solid strip of the plate having a width equal to p and thickness t is pt, therefore its resistance to tearing is

$$R = ptU_t.$$

If we now take a unit strip of the joint, due to the rivet hole the sectional area of the plate is reduced by dt, therefore the resistance to tearing now is

$$R_t = (p-d)tU_t.$$

Since in the length of joint equal to p there is one rivet only the shearing resistance is

$$R_s = \frac{\pi}{4} d^2 U_s.$$

Although the pressure between rivet and plate is distributed over a semi-cylindrical area it is customary to assume that the result is that of a uniformly distributed stress on an area equal to dt for each rivet, which is the projected area of the cylindrical surface, then

$$R_c = dtU_c.$$

From the above equations we obtain the efficiencies

$$E_t = \frac{R_t}{R} \times 100 = \frac{p-d}{p} \times 100,$$

$$E_s = \frac{R_s}{R} \times 100 = \frac{\pi d^2 U_s}{4ptU_t} \times 100,$$

$$E_c = \frac{R_c}{R} \times 100 = \frac{dU_c}{pU_t} \times 100.$$

Double-riveted Lap Joint (Fig. 3-6). In this joint there are two rivets in the unit strip and the following equations may readily be established:

$$R = ptU_t,$$

$$R_t = (p-d)tU_t,$$

$$R_s = \frac{\pi}{2} d^2 U_s,$$

$$R_c = 2dtU_c.$$

The efficiencies obtained in the same manner as for single riveted lap joint are:

$$E_t = \frac{p-d}{p} \times 100,$$

$$E_s = \frac{\pi d^2 U_s}{2ptU_t} \times 100,$$

$$E_c = \frac{2dU_c}{pU_t} \times 100.$$

Triple-riveted Lap Joint. In this joint (Fig. 3-7) there are are 3 rivets in the unit strip. Proceeding as before we obtain

$$R = ptU_t,$$

$$R_t = (p-d)tU_t,$$

$$R_s = \tfrac{3}{4}\pi d^2 U_s,$$

$$R_c = 3dtU_c.$$

The efficiencies are

$$E_t = \frac{p-d}{p} \times 100,$$

$$E_s = \frac{3}{4} \frac{\pi d^2 U_s}{ptU_t},$$

$$E_c = \frac{3dU_c}{pU_t}.$$

Another form of triple-riveted lap joint has the pitch of the outer rows of rivets twice that of the inner row (Fig. 3–7*a*). In this joint there are four rivets in the unit strip:

$$R = ptU_t,$$

$$R_t = (p-d)tU_t,$$

$$R_s = \pi d^2 U_s,$$

$$R_c = 4dtU_c.$$

Another method of failure is possible with this joint, viz.: a combination of tearing of plate and shearing of rivets. Thus the plate may tear at the center row of rivets and before failure could occur one of the outer rows of rivets would have to be sheared. For this case

$$R_{ts} = (p-2d)tU_t + \frac{\pi}{4} d^2 U_s.$$

The efficiencies therefore are

$$E_t = \frac{p-d}{p} \times 100,$$

$$E_s = \frac{\pi d^2 U_s}{ptU_t} \times 100,$$

$$E_c = \frac{4dU_c}{ptU_t} \times 100,$$

$$E_{ts} = \left(\frac{p-2d}{p} + \frac{\pi d^2 U_s}{4ptU_t}\right) \times 100.$$

Example. What is the efficiency of a double-riveted lap joint, diameter of rivets $\frac{3}{4}$ in., thickness of plate $\frac{3}{8}$ in., pitch $2\frac{1}{2}$ in.? Rivets and plates are of steel and $U_t = 60{,}000$, $U_s = 45{,}000$, $U_c = 80{,}000$.

$$E_t = \frac{p-d}{p} \times 100 = \frac{2.5 - .75}{2.5} \times 100 = 70 \text{ per cent},$$

$$E_s = \frac{\pi d^2 U_s}{2ptU_t} \times 100 = \frac{\pi \times .75^2 \times 45000}{2 \times 2.5 \times .375 \times 60000} \times 100 = 70\tfrac{1}{2} \text{ per cent},$$

$$E_c = \frac{2dU_c}{pU_t} \times 100 = \frac{2 \times .75 \times 80000}{2.5 \times 60000} \times 100 = 80 \text{ per cent}.$$

Actual efficiency is therefore 70 per cent.

Single-riveted Butt Joint (Fig. 3–5). In this joint the rivets are in double shear. There is one rivet on each side of the joint in the unit strip.

$$R = ptU_t,$$

$$R_t = (p-d)tU_t,$$

$$R_s = \frac{\pi}{4} d^2 U_s \times 1.8,$$

$$R_c = dtU_c,$$

$$E_t = \frac{p-d}{p} \times 100,$$

$$E_s = \frac{\pi d^2 U_s \times 1.8}{4ptU_t} \times 100,$$

$$E_c = \frac{dU_c}{pU_t} \times 100.$$

Double-riveted Butt Joint (Fig. 3–8). In this joint there are two rivets in double shear in the unit strip. All rivets are in double shear.

$$R = ptU_t,$$

$$R_t = (p-d)\,U_t,$$

$$R_s = \frac{\pi}{2} d^2 U_s \times 1.8,$$

$$R_c = 2dtU_c,$$

$$E_t = \frac{p-d}{p} \times 100,$$

$$E_s = \frac{1.8\pi d^2 U_s}{2ptU_t} \times 100,$$

$$E_c = \frac{2dU_c}{tU_t} \times 100.$$

Triple-riveted Butt Joint (Fig. 3–9). In this joint the pitch of the outer rows of rivets is twice that of the inner rows.

$$R = ptU_t,$$

$$R_t = (p-d)tU_t,$$

$$R_s = (4\times 1.8+1)\frac{\pi}{4}d_2U_s = 2.05\pi d^2U_s,$$

$$R_c = 5dtU_c.$$

Another method of failure possible is tearing of plate at middle row and shearing of outer row of rivets. For this case

$$R_{ts} = (p-2d)tU_t + \frac{\pi}{4}d^2U_s.$$

The efficiencies are

$$E_t = \frac{p-d}{p}\times 100,$$

$$E_s = \frac{2.05\pi d^2U_s}{ptU_t}\times 100,$$

$$E_c = \frac{5dU_c}{pU_t}\times 100,$$

$$E_{ts} = \left(\frac{p-2d}{p} + \frac{\pi d^2U_s}{4ptU_t}\right)\times 100.$$

EXAMPLE. Find the efficiency of a triple-riveted butt joint, Fig. 3–9. The pitch p is $7\frac{1}{2}$ ins., thickness of plate $\frac{1}{2}$ in., and diameter of rivet holes 1 in.

$$E_t = \frac{p-d}{p}\times 100 = \frac{7.5-1}{7.5}\times 100 = 87 \text{ per cent;}$$

$$E_s = \frac{2.05\pi d^2U_s\times 100}{p+U_t} = \frac{2.05\times\pi\times 1\times 45000\times 100}{7.5\times .5\times 60000} = 129\%,$$

$$E_c = \frac{5dU_c}{ptU_t}\times 100 = \frac{5\times 1\times 80000}{7.5\times .5\times 60000} = 89\%,$$

$$E_{ts} = \left(\frac{7.5-2}{7.5} + \frac{\pi\times 45000}{4\times 7.5\times .5\times 60000}\right)100 = 89\%.$$

The actual efficiency of the joint is therefore 87 per cent.

Design of Riveted Joints. From the preceding it is evident that the efficiency of a joint depends on three variables, thickness of plate t, diameter of rivets d, and the pitch p. Practice has fixed the size of the rivets to be used with any thickness of plate and the following equation will give good results for boiler joints:

$$d=1.5\sqrt{t}-\tfrac{3}{16}\text{ in.}$$

In designing a riveted joint it is usual to make its resistance to tension equal to its shearing resistance, or

$$R_t=R_s.$$

From this equation we may obtain the pitch for any type of joint. Thus for a single-riveted lap joint

$$R_t=R_s,$$

$$(p-d)tU_t=\frac{\pi}{4}d^2U_s;$$

and

$$p=\frac{\pi d^2U_s}{4tU_t}+d.$$

EXAMPLE. Design a double-riveted butt joint for plate $\tfrac{3}{8}$ in. in thickness.

$$d=1.5\sqrt{.375}-\tfrac{3}{16}$$
$$=.74=\tfrac{3}{4}\text{-in. rivet.}$$

This is the size of hole in plate or rivet diameter after it is driven.

$$R_t=R_s,$$

$$(p-d)tU_t=\frac{1.8\pi d^2U_s}{2},$$

$$p=\frac{.9\pi d^2U_s}{tU}+d,$$

$$=\frac{.9\pi(\tfrac{3}{4})^2 45000}{\tfrac{3}{8}\times 60000}+\tfrac{3}{4},$$

$$=3\tfrac{15}{16}\text{ in.}$$

However, in the design of boiler and similar joints it must be kept in mind that the maximum pitch is determined by the possibility of making a tight joint and this will often modify the calculated pitch. The minimum pitch should not be less than $3d$ in order to drive the rivets and form the head readily.

TABLES SHOWING DETAILS OF RIVET LAPS FOR DIFFERENT THICKNESSES OF BOILER PLATE AS ADVOCATED BY THE HARTFORD STEAM BOILER INSPECTION AND INSURANCE CO. FOR

DOUBLE RIVETED BUTT JOINTS

Thickness of Plate.	Diameter of Rivets.	Pitch of Rivets in Inches.	Width of Outside Butt Strap.	Width of Inside Butt Strap.	Thickness of Covering Straps.	Vertical or Transverse Pitch.	Edge of Butt Strap to Center of Rivets.	Pitch of Rivets Girth Seam.	Edge of Plate to Center of Rivets Girth Seam.	Strength of Joint.
In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	%
$\frac{5}{16}$	$\frac{11}{16}$	$2\frac{1}{4}\times4\frac{1}{2}$	$4\frac{1}{2}$	9	$\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{1}{8}$	$2\frac{1}{8}$	$1\frac{1}{8}$	83
$\frac{3}{8}$	$\frac{3}{4}$	$2\frac{3}{4}\times4\frac{3}{4}$	$4\frac{7}{8}$	$9\frac{7}{8}$	$\frac{5}{16}$	$2\frac{7}{16}$	$1\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{7}{32}$	82.9
$\frac{7}{16}$	$\frac{13}{16}$	$2\frac{15}{32}\times4\frac{15}{16}$	$5\frac{1}{4}$	$10\frac{1}{2}$	$\frac{3}{8}$	$2\frac{3}{8}$	$1\frac{15}{16}$	$2\frac{1}{4}$	$1\frac{7}{32}$	82
$\frac{1}{2}$	$\frac{7}{8}$	$2\frac{9}{16}\times5\frac{1}{8}$	$5\frac{5}{8}$	$11\frac{1}{4}$	$\frac{7}{16}$	$2\frac{13}{16}$	$1\frac{13}{32}$	$2\frac{1}{4}$	$1\frac{13}{32}$	80

TRIPLE RIVETED BUTT JOINTS

Thickness of Plate.	Diameter of Rivets.	Pitch of Rivets in Inches.	Width of Outside Butt Strap.	Width of Inside Butt Strap.	Thickness of Covering Straps.	Vertical or Transverse Pitch.	Edge of Butt Strap to Center of Rivets.	Pitch of Rivets Girth Seam.	Edge of Plate to Center of Rivets Girth Seam.	Strength of Joint.
In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	%
$\frac{1}{4}$	$\frac{1}{4}$	$2\frac{1}{4}\times4\frac{1}{2}$	$6\frac{1}{2}$	$11\frac{3}{8}$	$\frac{1}{4}$	$1\frac{1}{2}$	$\frac{27}{32}$	$2\frac{1}{16}$	$1\frac{1}{8}$	87.5
$\frac{9}{32}$	$\frac{9}{16}$	$2\frac{9}{32}\times4\frac{9}{16}$	$6\frac{3}{4}$	$12\frac{3}{8}$	$\frac{1}{4}$	$1\frac{1}{2}$	$\frac{15}{16}$	$2\frac{1}{16}$	$1\frac{1}{8}$	86
$\frac{5}{16}$	$\frac{11}{16}$	$3\frac{1}{8}\times6\frac{1}{4}$	$9\frac{1}{4}$	14	$\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{1}{4}$	$2\frac{1}{16}$	$1\frac{1}{8}$	88
$\frac{11}{32}$	$\frac{11}{16}$	$3\frac{1}{8}\times6\frac{1}{4}$	$9\frac{1}{4}$	14	$\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{1}{4}$	$2\frac{1}{8}$	$1\frac{7}{32}$	88
$\frac{3}{8}$	$\frac{3}{4}$	$3\frac{1}{4}\times6\frac{1}{2}$	$9\frac{3}{4}$	$14\frac{1}{4}$	$\frac{5}{16}$	$2\frac{3}{16}$	$1\frac{7}{32}$	$2\frac{1}{8}$	$1\frac{7}{32}$	87.5
$\frac{13}{32}$	$\frac{3}{4}$	$3\frac{1}{4}\times6\frac{1}{2}$	$9\frac{1}{4}$	$14\frac{1}{4}$	$\frac{5}{16}$	$2\frac{3}{16}$	$1\frac{7}{32}$	$2\frac{1}{8}$	$1\frac{7}{32}$	87.5
$\frac{7}{16}$	$\frac{7}{8}$	$3\frac{3}{8}\times6\frac{3}{4}$	$10\frac{1}{8}$	$15\frac{5}{8}$	$\frac{3}{8}$	$2\frac{1}{4}$	$1\frac{13}{32}$	$2\frac{7}{16}$	$1\frac{1}{2}$	86
$\frac{15}{32}$	$\frac{7}{8}$	$3\frac{1}{2}\times7$	$10\frac{3}{8}$	16	$\frac{13}{32}$	$2\frac{3}{8}$	$1\frac{13}{32}$	$2\frac{7}{16}$	$1\frac{1}{2}$	86
$\frac{1}{2}$	$\frac{15}{16}$	$3\frac{3}{4}\times7\frac{1}{2}$	11	$16\frac{3}{4}$	$\frac{7}{16}$	$2\frac{3}{8}$	$1\frac{17}{32}$	$2\frac{1}{2}$	$1\frac{1}{2}$	86.6

The butt straps are usually somewhat thinner than the plates to be joined. Generally their thickness is $\frac{1}{16}$ to $\frac{1}{8}$ in. less than that of the plates. In triple and quadruple riveted butt joints the butt straps are often of different widths, as shown in Fig. 3–9. Sometimes, especially in marine work, the edges are scalloped, as shown in Fig. 3–10. Both these devices enable the use of a wide pitch in the outer rows of rivets and at the same time permit the calking of the joint.

PROBLEMS

1. In a single-riveted lap joint the thickness of plate is $\frac{3}{8}$ in., the diameter of rivet is $\frac{3}{4}$ in., and the pitch is $2\frac{1}{8}$ in. Find the efficiency of this joint.

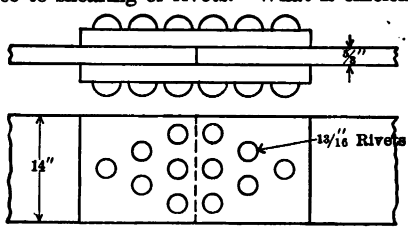
2. Design a double-riveted lap joint making shearing resistance equal to tearing resistance. The plate is $\frac{1}{2}$ in. thick and rivets are $\frac{7}{8}$ in. diameter.

aver.?

3. In a triple-riveted lap joint the pitch of the inner rows of rivets is $2\frac{3}{4}$ in. The outer row of rivets is of double the pitch. The rivets are 1 in. diameter and the plates are $\frac{5}{8}$ in. thick. Find the efficiency of this joint.

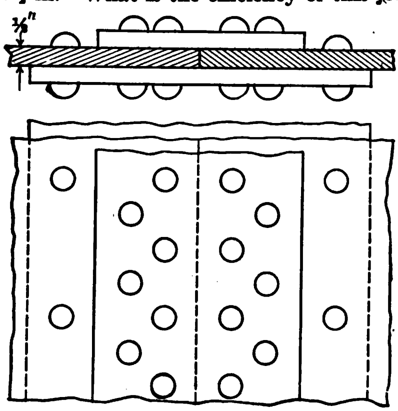
4. Design a double-riveted butt joint for $\frac{3}{8}$-in. plate. Find efficiency of joint.

5. Sketch shows a method of joining two plates frequently used in structural work. Find resistance of joint to tearing at outer row of rivets, at second row, and resistance to shearing of rivets. What is efficiency of this joint?



Chap. III. Prob. 5.

6. Design a triple-riveted butt joint of the form shown in sketch. The thickness of plate is $\frac{1}{2}$ in. What is the efficiency of this joint?



Chap. III. Prob. 6.

7. In a double-riveted butt joint the outer row of rivets is in single shear and of double the pitch of inner row. The plate is $\frac{5}{8}$ in. thick, the rivets 1 in. diameter and pitch of outer row is 5 in. Determine efficiency of joint.

8. Design a triple-riveted butt joint for $\frac{5}{8}$-in. plate. All rivets are in double shear and of same pitch. Find efficiency of this joint.

CHAPTER IV

KEYS AND COTTERS

Kinds of Keys. Keys are rectangular or round bars, the purpose of which is to prevent relative rotation of two parts, such as a shaft and pulley. The *sunk* key shown in Fig. 4–1 is the commonest type. For light work the *flat* key, Fig. 4–2, and the *saddle* key, Fig. 4–3, are sometimes used. When the hub is very light,

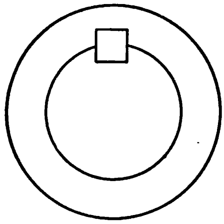


FIG. 4–1.

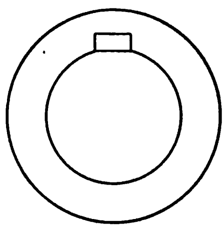


FIG. 4–2.

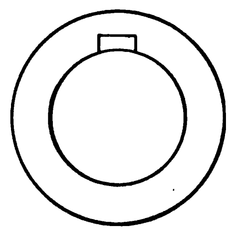


FIG. 4–3.

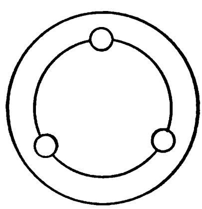


FIG. 4–4.

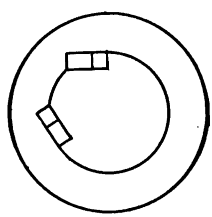


FIG. 4–5.

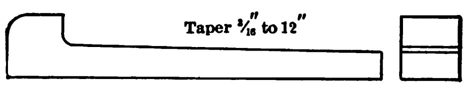


FIG. 4–6.

round or *pin* keys may be used as in Fig. 4–4. For very heavy work, such as the belt wheels of steam engines, two *tangential* keys are often used, as shown in Fig. 4–5. To facilitate removal of keys they are frequently provided with gib heads, Fig. 4–6. To obtain a tight fitting key the top is tapered from $\frac{1}{8}$ to $\frac{1}{4}$ in.

per foot of length. When the key prevents relative rotation but permits motion lengthwise it is called a *feather* key or *spline.* In this case the sides of the key are parallel and it is fastened by screws or otherwise (Fig. 4–7 and 4–8) to one of the two parts it connects and is an easy fit in the other.

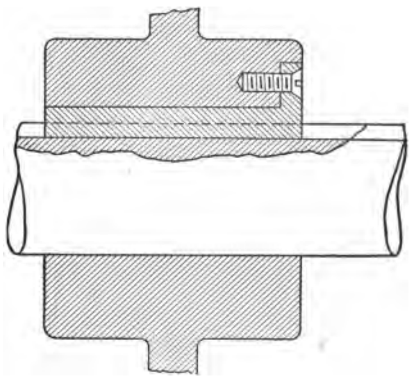


FIG. 4–7.

Stresses in Keys. The twisting moment which a key transmits produces shearing stresses in the plane CD (Fig. 4–9) and crushing on sides of key.

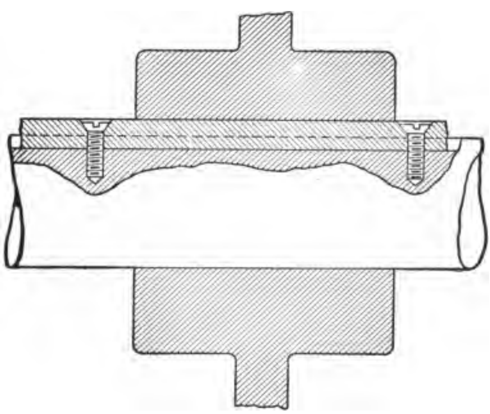


FIG. 4–8

Let $T=PR=$twisting moment to be transmitted,

$l=$length of key.

Since the moment of the shearing stress and of the compressive stress about center of shaft must each equal the twisting moment to be transmitted,

$$bls_s\frac{d}{2}=T, \quad \ldots\ldots\ldots \quad (1)$$

and

$$\tfrac{1}{2}tls_c\frac{d}{2}=T; \quad \ldots\ldots\ldots \quad (2)$$

then equating (1) and (2),

$$bs_s=\tfrac{1}{2}ts_c,$$

and

$$t=2b\frac{s_s}{s_c}. \quad \ldots\ldots\ldots \quad (3)$$

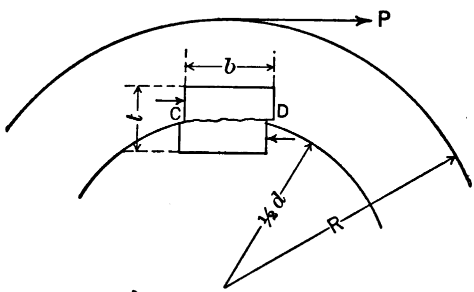


FIG. 4-9.

If a key fits tightly in the key seat the crushing resistance is considerably increased and a value of $\frac{1}{4}$ to $\frac{1}{2}$ may be assumed for $\frac{s_s}{s_c}$, for steel keys. This makes $t=\frac{1}{2}b$ to $t=b$; the latter is a square key which is used by manufacturers of transmission machinery.

It is usually assumed that the key should be capable of transmitting the entire twisting moment of the shaft upon which it is placed, then

$$bls_s\frac{d}{2}=\frac{\pi}{16}d^3s_s\,^*,$$

* This is the twisting moment which can be transmitted by a shaft of diameter $=d$, see Chapter VI.

and

$$bl=\frac{\pi}{8}d^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

A common value of b is $\frac{1}{4}d$ and substituting this in (4) we obtain

$$l=\frac{\pi}{2}d \text{ as the minimum length of key.}$$

Manufacturers have not adopted standard dimensions for keys, and both square and rectangular keys are used. The following tables taken from catalogs of prominent transmission machinery firms give good average practice:

TABLE I

SQUARE KEYS

Shaft Dia.	Width.	Thickness.	Shaft Dia.	Width.	Thickness.
$1\frac{1}{4}$	$\frac{7}{32}$	$\frac{7}{32}$	$2\frac{1}{2}$	$\frac{9}{16}$	$\frac{9}{16}$
$1\frac{1}{2}$	$\frac{5}{16}$	$\frac{5}{16}$	3	$\frac{11}{16}$	$\frac{11}{16}$
$1\frac{3}{4}$	$\frac{7}{16}$	$\frac{7}{16}$	$3\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$
2	$\frac{7}{16}$	$\frac{7}{16}$	4	$\frac{7}{8}$	$\frac{7}{8}$

TABLE II

RECTANGULAR KEYS

Shaft Dia.	Width.	Thickness.	Shaft Dia.	Width.	Thickness.
$1\frac{1}{4}$	$\frac{5}{16}$	$\frac{5}{16}$	$3\frac{1}{2}$	$\frac{7}{8}$	$\frac{1}{2}$
$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	4	1	$\frac{5}{8}$
$1\frac{3}{4}$	$\frac{7}{16}$	$\frac{9}{32}$	5	$1\frac{1}{8}$	$\frac{11}{16}$
2	$\frac{1}{2}$	$\frac{5}{16}$	6	$1\frac{3}{8}$	$\frac{13}{16}$
$2\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{8}$	7	$1\frac{1}{2}$	$\frac{7}{8}$
3	$\frac{3}{4}$	$\frac{7}{16}$			

Cotters. A cotter is a form of key in which the forces tending to separate the parts connected act at right angle to their axis. Figs. 4–10 to 4–13 show some of the common forms of cottered

joint. In Fig. 4–11 a gib is used with the cotter. This prevents the ends from spreading when the cotter is driven. It also

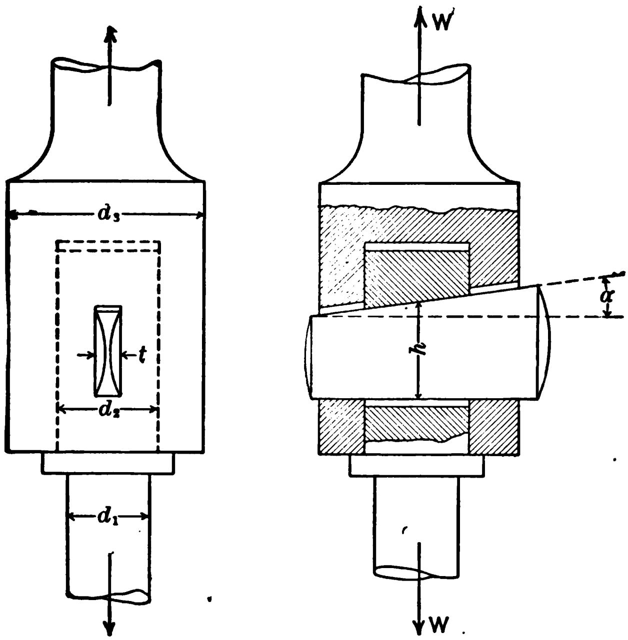


Fig. 4–10.

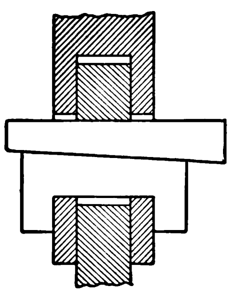


Fig. 4–11.

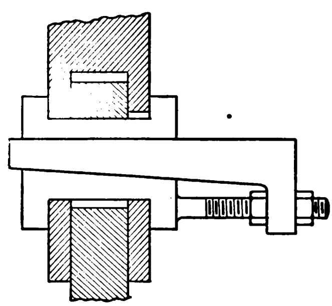


Fig. 4–12.

simplifies the construction in avoiding a tapered hole. Fig. 4–12 shows a cotter with double gib and means for locking cotter. In Fig. 4–13 the cotter has a double taper with a locking device,

The Strength of Joint. In Fig. 4–10 let a tensile load of W lbs. be applied to rod, then its diameter is

$$d_1=\sqrt{\frac{W}{.785s_t}}.$$

To make the rod through the cotter (Fig. 4–14) of equal strength it must have an equal cross-sectional area, that is

$$\frac{\pi}{4}d_1^2=\frac{\pi}{4}d_2^2-d_2t.$$

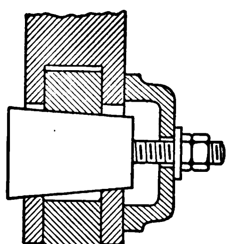


FIG. 4–13.

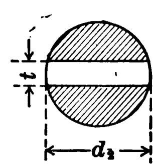


FIG. 4–14.

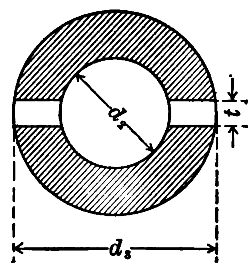


FIG. 4–15.

The thickness, t, of cotter may be expressed in terms of d_2, it being usually from $\frac{1}{4}d_2$ to $\frac{1}{3}d_2$, or say kd_2, then

$$\frac{\pi}{4}d_1^2=\frac{\pi}{4}d_2^2-kd_2^2,$$

and

$$d_1^2=\left(1-\frac{4k}{\pi}\right)d_2^2;$$

$$\therefore\quad d_2=\frac{d_1}{\sqrt{1-1.27k}}. \qquad \ldots\ldots \qquad (5)$$

For the two extreme values $k=\frac{1}{4}$ and $k=\frac{1}{3}$ this reduces to

$$d_2=1.22d_1 \text{ to } 1.31d_1.$$

The socket at its weakest point has a section as in Fig. 4–15. This section is subjected to tension and from equation

$$W=As_t$$

we obtain

$$\frac{W}{s_t}=A=\frac{\pi}{4}(d_3^2-d_2^2)-t(d_3-d_2). \quad . \quad . \quad . \quad . \quad (6)$$

This equation may be solved for d_3, or better, the outside diameter may be assumed and equation (6) solved for stress s_t.

The cotter is in double shear as shown in Fig. 4–16, then

$$W=As_s,$$

$$A=2ht=\frac{W}{s_s},$$

$$\therefore \quad h=\frac{W}{2ts_s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The cotter is also subjected to compression, the compressive stress is

$$s_c=\frac{W}{td_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

To prevent shearing out of the end of rod and of socket the cotter should be placed in from these ends a distance at least equal to $\frac{1}{2}h$ and preferably $\frac{3}{4}h$.

The taper of the cotter is usually from $\frac{1}{2}$ in. in 12 ins. to 1 in. in 12 ins. To obtain a self-locking cotter, that is, one in which the cotter will not loosen no matter how large the force W, the angle α (Fig. 4–10) must be such that tan $\alpha<\mu$; where μ is the coefficient of friction between the surface in contact. Where the joint is subjected to a live or variable load (connecting rods) the cotter is prevented from loosening by a set screw.

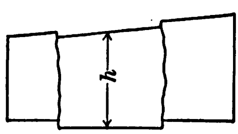


Fig. 4–16.

PROBLEMS

1. Find breadth of a key which is to transmit 20 H.P. at 120 R.P.M., the length of key is 3 ins. The stress not to exceed 8000 lbs. per square inch. If thickness of key is $\frac{1}{2}$ of breadth plus $\frac{1}{8}$ in. what is the compression stress. The diameter of shaft is $2\frac{1}{2}$ in.

2. A pulley is keyed to a shaft 3 ins. in diameter by a key 6 ins. long. Find breadth and thickness of key such that strength of key is equal to strength of shaft. The stresses to be $s_s = 9000$ and $s_c = 20,000$.

3. A lever 40 ins. long is keyed to a shaft 3 ins. diameter. A load of 1000 lbs. acts at end of lever. Design the key.

4. Design a cottered joint to fasten a piston rod to the cross-head. The diameter of engine cylinder is 30 ins. and the steam pressure is 150 lbs. per square inch. Assume steel for all parts, making $s_t = 8000$ lbs., $s_s = 6000$. Make thickness of cotter (t).3 of diameter of rod at point where it is placed.

5. In a hydraulic press the top platen is held by four rods cottered into base of machine. The ram of press is 12 in. diameter and the maximum pressure is 600 lbs. per square inch. Design the cotter joint making all necessary assumptions.

6. Two steel rods $2\frac{1}{2}$ ins. diameter are connected by means of a cottered joint at their ends. The socket formed at the end of one rod is 6 ins. outside diameter and 3 ins. inside diameter. The cotter is $\frac{3}{4}$ in. thick and has a depth of $3\frac{3}{4}$ in. Determine how failure of this joint would occur.

CHAPTER V

SHRINK AND FORCE FITS

When two pieces have to be fastened together very firmly shrink or force fits are often employed. The elastic stresses produced in the parts are the forces which hold them together. In shrink fits one of the members to be fastened together is heated and in this condition it fits over the other member; upon cooling it contracts, thus gripping the inner one firmly. In force fits the outer member is made slightly smaller than the inner one and forced over it by heavy pressure.

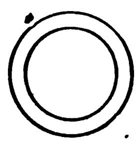


Fig. 5-1.

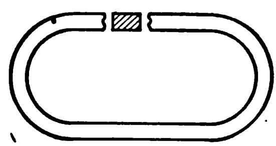


Fig. 5-2.

Shrink links are frequently used to hold the halves of a large gear or fly-wheel together. Their forms are shown in Figs. 5-1 to 5-5. Take the case of link in Fig. 5-4 and let

A = sectional area,
l = length before heating,
s_t = tensile stress per square inch,
E = modulus of elasticity,
λ = total elongation due to heating,
T = total tension in link,
α = coefficient of expansion,
t = theoretical temperature increase necessary;

then

$$E = \frac{\text{stress}}{\text{strain}} = \frac{s_t}{\frac{\lambda}{l}},$$

$$s_t = E\frac{\lambda}{l},$$

$$T = As_t = AE\frac{\lambda}{l};$$

and

$$\lambda = \alpha t l,$$

$$\therefore \quad t = \frac{\lambda}{\alpha l}.$$

In the above the compression produced in the parts held together has been neglected, which may be safely done, as it is extremely small in all cases where such links are used.

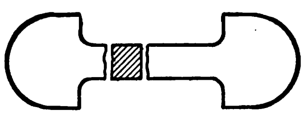


FIG. 5–3.

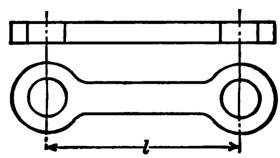


FIG. 5–4.

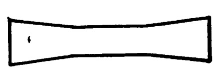


FIG. 5–5.

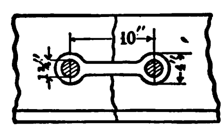


FIG. 5–6.

EXAMPLE. A crack in the frame of a machine is to be repaired by means of a shrink link as shown in Fig. 5–6. Assume steel links and bolts, $E = 30{,}000{,}000$ and $\lambda = .01$ in., $A = 2$ sq.in., then

$$s_t = E\frac{\lambda}{l} = \frac{30000000 \times .01}{10}$$

$$= 30{,}000 \text{ lbs. per square inch;}$$

and

$$T = As_t = 60{,}000 \text{ lbs.}$$

The bolts are in shear and the stress is

$$s_s = \frac{T}{\frac{\pi}{4}d^2} = \frac{60000}{2.4}$$

$$= 25{,}000 \text{ lbs. per square inch.}$$

In practice the grade of workmanship and finish of surfaces fitted greatly affect the shrinkage allowance and the values as calculated above would be too small for any but the highest grade of work where accuracy is possible.

Allowance for Force and Shrink Fits. Crank pins are frequently fitted into their cranks by shrink or force fits; built-up crank-shafts such as are used for marine engines are assembled in the same way; car wheels are fitted to their axles by force fits, and the steel tires of locomotive wheels are shrunk on. The difference in diameters of the two surfaces fitted together is termed the allowance. This varies greatly in practice but it may be expressed in the form

$$x=\frac{k}{\sqrt{d}}.$$

Where x is the allowance in thousandths of an inch per inch of diameter, k is a constant and d is the diameter of the parts fitted. For shrink fits a value of $k=4$ may be used and for force fits $k=6$ gives results agreeing well with general practice. It must be understood, however, that the value of k varies with the class of work and the finish and accuracy of surfaces.

EXAMPLE. A crank web is to be shrunk on a crank-pin 9 ins. diameter; what is the diameter of hole in web?

$$x=\frac{4}{\sqrt{9}}=\frac{1.33}{1000}\text{ in.,}$$

$$\text{Total allowance}=9\times\frac{1.33}{1000}=\frac{12}{1000}\text{ in.,}$$

$$\text{Diameter of hole}=9-.012=8.988\text{ in.}$$

CHAPTER VI

SHAFTS AND AXLES

A bar which turns about its axis and transmits power is termed a shaft. Formerly wrought iron was the favorite material used for shafting, but on account of lower cost and greater strength steel is now used almost exclusively. For ordinary shafting mild steel is employed. Where the work is very heavy, and weight is to be kept low, alloy steels are frequently used. Thus in motor cars and in marine engines vanadium and nickel steels are used. Commercial shafting is either turned or cold-rolled. Turned shafting is made by turning down bars of steel made from the ingot by hot-rolling. It is $\frac{1}{16}$ in. less in diameter than its "nominal size." Thus 2-in. shafting is $1\frac{15}{16}$-in. diameter. Cold-rolled shafting is made from the same material as the turned shafting, but the diameter is reduced by rolling cold under great pressure, or it is drawn through dies which reduce its diameter, when it is known as cold-drawn shafting.

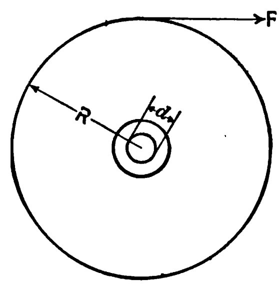


FIG. 6–1.

Shafting Subjected to Torsion Only. If a shaft carries no gears or pulleys and transmits power it will be subjected to torsional stress only.

Let H = horse-power to be transmitted,
N = revolutions per minute of shaft,
T = torque in inch-pounds,
d = diameter of shaft in inches,
P = tangential force, at radius R inches, to be transmitted. (Fig. 6–1.)

From equation (15) chapter I,

$$T = s_s z_p;$$

but for a circular section $z_p = \frac{\pi}{16}d^3$, then

$$T = \frac{\pi}{16}d^3 s_s. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Solving this equation for d we obtain

$$d = 1.72\sqrt[3]{\frac{T}{s_s}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Generally either the tangential force P or the horse-power H is known. The value of T in terms of H is derived as follows:

$$\text{Horse-power} = \frac{\text{Work done per minute in foot-pounds}}{33000}$$

$$= \frac{P \times \text{velocity in feet}}{33000}$$

$$= \frac{P \times 2\pi \frac{R}{12} N}{33000};$$

but

$$PR = T;$$

then

$$H = \frac{2\pi TN}{12 \times 33000},$$

$$T = \frac{63025H}{N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If this value of T be now substituted in equation (2) the result is

$$d = 68.5\sqrt[3]{\frac{H}{s_s N}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hollow Shafts. The stress produced by torsion in any section of a shaft is not uniformly distributed over that section, but varies directly as the distance from the center of the shaft. Then the stress is a maximum at the outside or " skin " of the shaft and is zero at the center. It is evident, therefore, that the material near center of shaft does not add much to its strength. For this reason a considerable saving in weight may be effected by the

use of a hollow shaft, and large shafts, such as are used in marine work, are frequently made hollow. If D is the outside diameter of a hollow shaft and d the diameter of hole the section modulus is

$$z_p = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right),$$

and the torque which such a shaft can transmit is

$$T = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)s_s. \quad \dots\dots\dots \quad (5)$$

If $\dfrac{d}{D} = k$ and we substitute for d its value kD we obtain from (5)

$$D = 1.72\sqrt[3]{\frac{T}{s_s(1-k^4)}}. \quad \dots\dots \quad (6)$$

Example. What horse-power can a hollow steel shaft transmit when rotating at 120 R.P.M.? The outside diameter is 10 ins., inside diameter is 6 ins. and the stress is not to exceed 10,000 lbs. per square inch.

From (5)

$$T = .196\left(\frac{10000 - 1296}{10}\right)10000$$

$$= 1,706,000 \text{ inch-pounds.}$$

From (1)

$$H = \frac{TN}{63025} = \frac{1706000 \times 120}{63025}$$

$$= 3250.$$

Shafts Subjected to both Bending and Twisting. If a shaft carries a heavy belt wheel or gear it is subjected to both a twisting moment, due to power which is transmitted, and to a bending moment, due to weight of pulley or gear, and to belt pull or tooth pressure. In this case the ideal twisting moment is obtained (equation (17), chapter I).

$$T_i = M + \sqrt{T^2 + M^2};$$

and

$$d = 1.72\sqrt[3]{\frac{T_i}{s_s}}. \quad \dots\dots\dots\dots \quad (7)$$

EXAMPLE. In Fig. 6–2 the shaft transmits 40 H.P. at 90 R.P.M. by means of a gear 60 ins. diameter placed at center. What should diameter of shaft be so that the stress will not exceed 10,000 lbs. per square inch? The weight of gear is 800 lbs.

$$T=\frac{63025\times H}{N}$$

$$=\frac{63025\times 40}{90}=28{,}000 \text{ in.-lbs.}$$

The tooth pressure is

$$P=\frac{T}{R}=\frac{28000}{30}=930 \text{ lbs.}$$

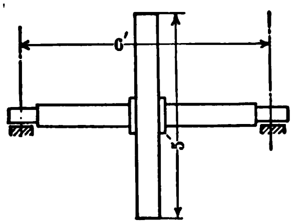


FIG. 6–2.

The total load on shaft is 800+930=1730 lbs., therefore the bending moment is

$$M=\frac{1730}{2}\times 36=31{,}200 \text{ in.-lbs.},$$

$$T_i=31200+\sqrt{28000^2+31200^2}$$

$$=73000 \text{ in.-lbs.};$$

and

$$d=1.72\sqrt[3]{\frac{73000}{10000}}$$

$$=3.34, \text{ say } 3\tfrac{3}{8} \text{ in.}$$

Practical Rules. In practice it is frequently impossible to predetermine the distribution of loads on a shaft and the diameter is calculated from the horse-power to be transmitted. The following equation may be used for steel shafts:

Head Shafts.

$$d=\sqrt[3]{\frac{100H}{N}}. \qquad (8)$$

Line Shafts.

$$d=\sqrt[3]{\frac{60H}{N}}. \qquad (9)$$

Counter Shafts.

$$d=\sqrt[3]{\frac{40H}{N}}. \qquad (10)$$

Proper support of the shafts is assumed in all cases. The torsional stresses corresponding to the above equations are 3200 lbs. per square inch for head shafts, 5350 lbs. per square inch for line shafts and 8000 lbs. per square inch for counter shafts.

The distance between center of bearings of line shafts depends on the diameter of the shaft, on the amount of transverse stress due to belt pull, etc., and to some extent on the speed of shaft. For average conditions the following table gives safe values of this distance.

Shaft dia.......	2″	2½″	3″	3½″	4″	4½″	5″
Center dist.....	9′ 6″	10′ 6″	11′ 0″	12′ 6″	14′ 0″	16′ 0″	18′ 0″

Torsional Rigidity of Shafts. If power be transmitted to a machine through a long line of shafting the angular distortion may have a disturbing effect on the proper running of same if there are sudden variations in load. It is therefore usual to limit this distortion, a common rule being that the angle through which the shaft may be twisted is not to exceed $\frac{1}{4}°$ for each 3 ft. of length. From equations (16) Chapter I:

$$\delta=\frac{57.2TL}{I_pE_s}. \qquad (11)$$

Substituting for δ its value $\frac{1}{4}°$ and for I_p, $\frac{\pi}{32}d^4$, for E_s 13,000,000 (steel) and L, 36 ins. we obtain

$$\frac{1}{4}=\frac{57.2T\times 36}{\frac{\pi}{32}d^4\times 13000000};$$

and

$$d = .378\sqrt[4]{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

Axles. An axle is a rotating or oscillating bar which does not transmit any torsional moment but is subjected to loads which produce bending stresses. Thus in Fig. 6-3 the axle carries a bell crank and the resultant R of the forces, P_1 and P_2, produce bending stresses in it. The axle therefore is designed as a beam of circular cross-section carrying a concentrated load R. It is to be noted that the material of the axle is subjected to a continually varying stress and therefore a correspondingly high factor of safety is to be used. In a rotating axle, for instance car axles, the stress will vary from a maximum tension to a maximum compression.

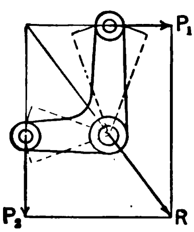


FIG. 6-3.

PROBLEMS

1. What horse-power may be transmitted by a shaft 2½ ins. diameter at 140 R.P.M. if the maximum stress is 10,000 lbs. per square inch? Shaft is subjected to torsion only.

2. Find the diameter of a shaft which will transmit a torque due to 400 lbs. acting at a radius of 18 in. The stress not to exceed 6000 lbs. per square inch.

3. A shaft 3 in. diameter transmits a torque of 16,000 in.-lbs., the ultimate shear resistance of the shaft material being 45,000 lbs. per square inch. What is the factor of safety?

4. What horse-power may be transmitted by a hollow shaft at 90 R.P.M.? The outside diameter is 15 ins., the inside diameter is .6 of the outside diameter; the stress to be 8000 lbs. per square inch.

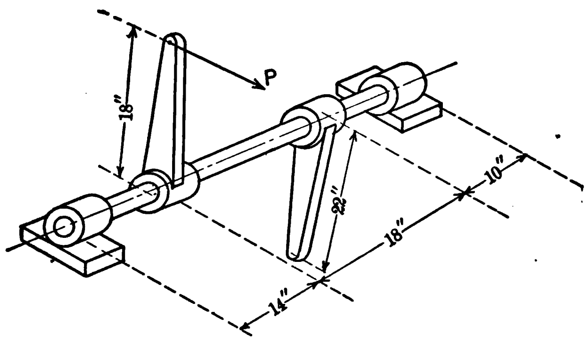
5. A hollow shaft of equal strength is to be substituted for a solid shaft 12 ins. in diameter. The inside diameter is ¾ of the outside diameter. Find diameter, allowing a stress of 9000 lbs. per square inch.

6. Compare the weight of a hollow shaft 18 ins. outside diameter and 10 ins. inside diameter with that of a solid shaft of equal strength.

7. A shaft is supported on two bearings 8 ft. between centers. It carries a pulley 3 ft. from one bearing. The load on shaft due to belt pull and to weight of pulley is 1100 lbs. The shaft transmits transmits 25 H.P. at 120 R.P.M. Find diameter of shaft, allowing a maximum stress of 8000 lbs. per square in.

8. What horse-power can a hollow steel shaft transmit, the outside diameter of which is 10 ins., inside 6 ins. and speed 120 R.P.M.? The safe stress is 10,000 lbs.

9. The force P (1500 lbs.) at lever is transmitted to lever B. Find diameter of shaft, safe stress is 8000 lbs. per square inch. Design the levers, assuming them to be of rectangular section, depth of rectangle three times the width and safe stress 10,000 lbs. per square inch.



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10. Find diameter of a line shaft carrying a fair proportion of pulleys. It transmits 60 H.P. at 150 R.P.M.

11. A shaft transmits 30 H.P. at 120 R.P.M. What is the total angle of twist if the shaft is 60 ft. in length and 2½ ins. in diameter?

12. A hollow steel shaft has an outside diameter of 9 ins. and inside diameter of 6 ins. Is transmits a torque of 300,000 in.-lbs. Find stress in shaft.

13. A shaft is supported on two bearings 10 ft. between centers. It carries two gears of 60 and 36 ins. diameter respectively, placed 3 ft. from each bearing. The shaft transmits 80 H.P. at 90 R.P.M. Determine the diameter of shaft, allowing a stress of 10,000 lbs. per square inch. The weight of gears is 900 lbs. and 500 lbs. each. Assume that the tooth pressure acts vertically downward.

CHAPTER VII

COUPLINGS AND CLUTCHES

Couplings. When a long line of shafting is to be installed it is necessary to connect the ends of the lengths which make up this line, as the usual length of a piece of shafting is 20 ft. to

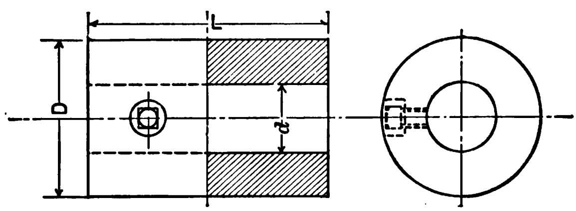


FIG. 7-1.

25 ft. The machine part used is called a shaft coupling. There are several different kinds of couplings used for this purpose.

The *sleeve* or *muff* coupling illustrated in Figs. 7-1 and 7-2 is simply a hollow sleeve keyed to the shaft ends, the first being a

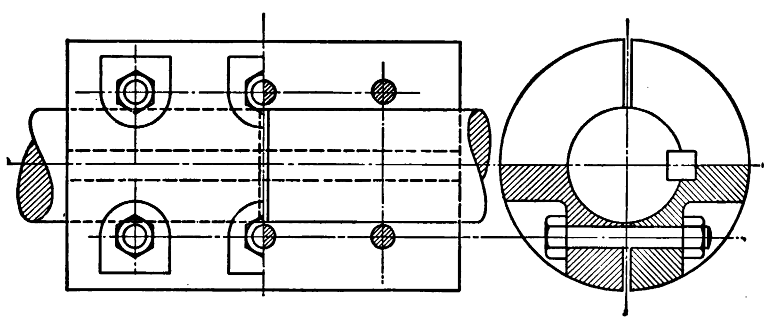


FIG. 7-2.

solid, while the latter is a split coupling. The length L, of these couplings may be made $3\frac{1}{2}d$ and the outside diameter $D=2d+1$ in.

For large shafts *flange* couplings are more commonly used. Fig. 7-3 illustrates this type. The following proportions may be

used: Outside diameter $D=3d+2$ in., length $L=3d$, bolt circle diameter $B=2\frac{1}{2}d$. The bolts are in shear and may be calculated on the assumption that they are capable of transmitting the

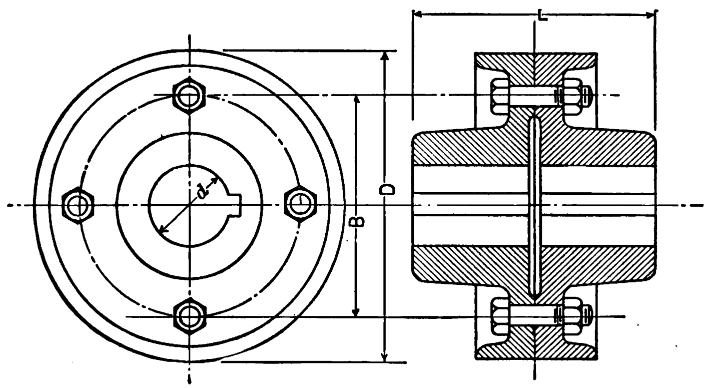


FIG. 7-3.

same torque, T, as the shaft, and that the load is carried by $\frac{1}{2}$ of the bolts.

Let d_1 = bolt diameter,

n = number of bolts.

Then

$$T=\frac{\pi}{16}d^3s_s=\frac{\pi}{4}d_1^2\frac{n}{2}s_s\frac{B}{2};$$

and

$$d_1=\sqrt{\frac{d^3}{nB}}; \quad \ldots \ldots \quad (1)$$

if

$$B=2\tfrac{1}{2}d;$$

then

$$d_1=\sqrt{\frac{d^3}{2.5nd}}=\frac{.63d}{\sqrt{n}}. \quad \ldots \ldots \quad (2)$$

The number of bolts may be 4 for shafts up to $3\frac{1}{2}$ ins. diameter, and 8 for shafts from 4 to 8 ins. diameter.

The **Sellers** coupling (Fig. 7-4), drives by means of friction between the conical bushings and the shaft. These bushings are split lengthwise and the bolts tighten them around the shaft, due to an equal taper on the inside of sleeve. The length L may

be $3d+2$ in. and the outside diameter D is $2d+2$ in. If R is the mean radius of the bushings the tangential force at this circle is

$$P=\frac{T}{R},$$

and if α is the angle of the bushings the axial force required is

$$Q=P\frac{\sin\alpha+\mu\cos\alpha}{\mu},$$

where μ is the coefficient of friction. The load on each bolt is therefore $\frac{1}{3}Q$. The angle α may be from 5° to 10°.

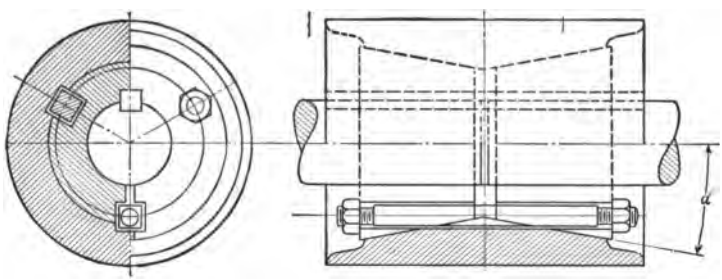


FIG. 7-4.

In all the above types of couplings accurate alignment of shafts is essential. These couplings should be placed very close to a bearing and as far from pulleys or gears as possible, to relieve them of bending stresses.

The **universal** coupling shown in Fig. 7-5 is used to connect two shafts which meet at an angle or where some flexibility is necessary as in motor cars. It should be remembered that with one coupling connecting two shafts which meet at an angle the transmission of motion will not be uniform and to obtain such motion of the driven shaft two couplings must be used.

A **flexible** coupling is advantageous where two shafts are to be coupled and accurate alignment cannot be maintained, or if a prime mover, such as an electric motor, is coupled directly to a shaft. Figs. 7-6 and 7-7 show two types of this coupling. In the first the connection is a band of leather or cotton, while the latter has a number of leather links connecting the two halves

of coupling. The outside diameter D may be made $5d$, the length of hubs L is $1\frac{3}{4}d$, and width w of band or links is d.

Clutches. Couplings which may be readily disengaged are usually termed clutches. These are used not only to connect two shafts but also to couple pulleys, gears, sprocket wheels, hoisting drums and similar machine parts to their shafts. The variaticns

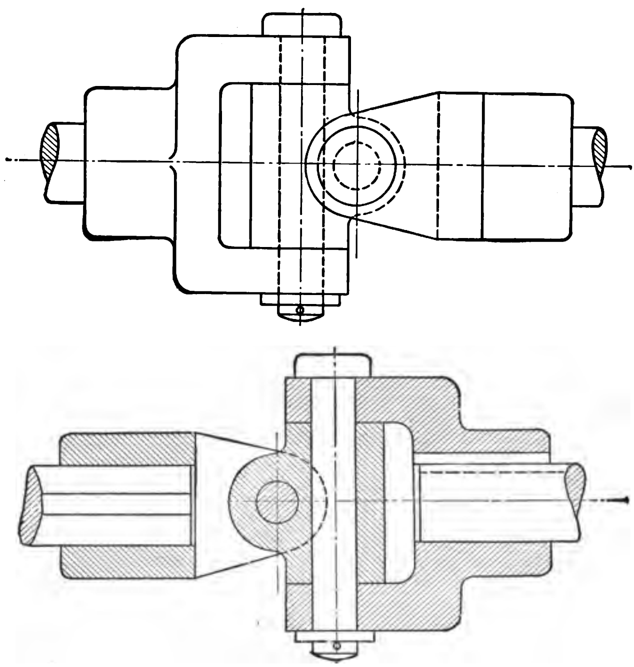


FIG. 7-5.

in construction are very numerous. They may, however, be divided into two classes, *positive* clutches and *friction* clutches, accordingly as the motion is transmitted by positive contact pressure or by friction between surfaces.

Positive Clutches. The *jaw clutch* shown in Figs. 7-8 and 7-8*a* is the commonest type of positive clutch. One part of this clutch is keyed tightly to its shaft while the other slides on a feather key and therefore can be moved axially until its teeth engage with those of the other half. Fig. 7-8 shows a clutch capable of transmitting rotation in both directions, while that

shown in Fig. 7-8*a* can do so in one direction only. Such clutches can be thrown into or out of action only at very low speeds. They are frequently used in conveying machinery. The sliding

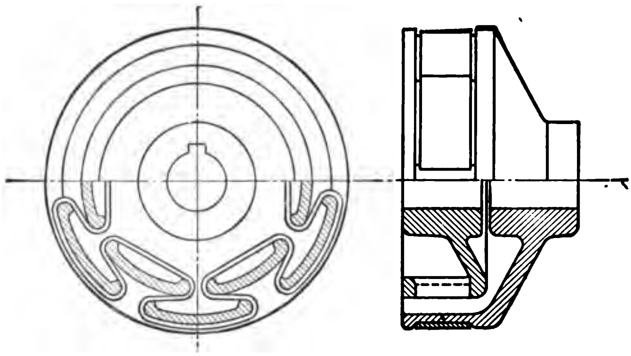


FIG. 7-6.

member for large clutches should have two feather keys placed 180° apart, otherwise it is difficult to move same along the shaft.

Pin clutches are of the positive type in which the motion is

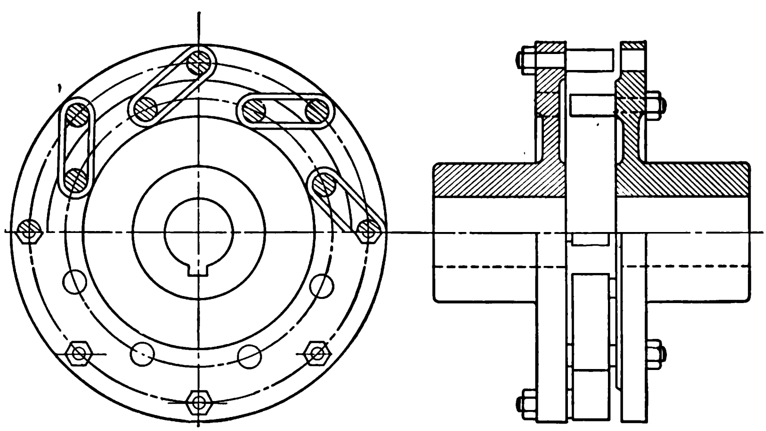


FIG. 7-7.

transmitted by means of a steel pin. They are largely used in punching and shearing machinery.

Friction Clutches. These may be divided into cone, disc, cylindrical and coil clutches, according to the surfaces in con-

tact. Figs. 7–9 to 7–12 show examples of each of these types. In all the motion is transmitted from the driving part to the driven by means of friction between two or more surfaces. These sur-

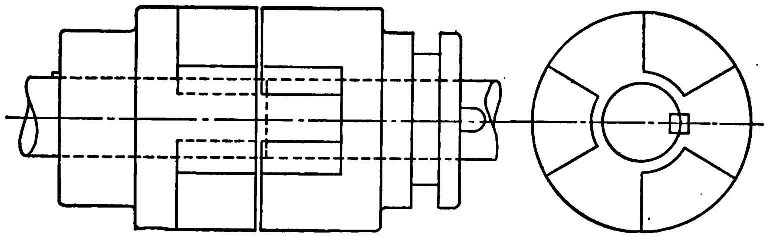


FIG. 7–8.

faces are pressed together with sufficient force so that the friction between them will overcome the resistance to motion offered by the driven member. The contact surfaces may be steel, cast iron, bronze, brass, wood, cork, leather or fiber.

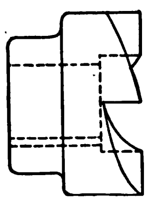


FIG. 7–8a.

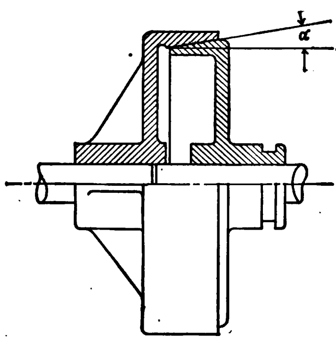


FIG. 7–9.

Cone Clutch. This clutch, illustrated in Fig. 7–9, consists of a cone which may be moved along the shaft until it comes in contact with the cup into which it fits.

Let P_t = the tangential force to be transmitted,
P_n = the normal pressure between friction surfaces,
P_a = axial force on cone,
R = mean radius of cone in inches,
μ = coefficient of friction,
H = horse-power to be transmitted,
N = revolutions per minute;

then

$$P_t = \frac{63025H}{RN},$$

but the friction between contact surfaces must be at least equal to the tangential force to be transmitted; then

$$P_n = \frac{P_t}{\mu},$$

$$P_a = P_n \sin \alpha$$

$$= P_t \frac{\sin \alpha}{\mu}. \quad \ldots \ldots \ldots \quad (3)$$

This would give the axial force required to transmit the tangential force P_t or to engage the clutch slowly, neglecting the friction which must be overcome in moving one surface over the other.

The angle α is made 8° to 12° and the normal pressure per square inch is 10 to 40 lbs., depending on the material of the contact surfaces and on the condition of service for which clutch is used. High speeds and soft materials require the lower unit pressures.

Example. Design a cone clutch to couple a pulley to a 2-in. shaft. This pulley transmits 10 H.P. at 200 R.P.M.

The mean radius of clutch may be taken at 4 to 5 times the diameter of shaft, say 8 ins., then

$$P_t = \frac{63025 \times 10}{8 \times 200} = 395 \text{ lbs.}$$

For wood on cast iron we may assume $\mu = .2$ and α may be 12°, then

$$P_n = \frac{P_t}{\mu} = \frac{395}{.2} = 1975 \text{ lbs.}$$

Let the allowable normal pressure be 20 lbs. per square inch, then the total surface of contact is

$$S = \frac{1975}{20} = 100 \text{ sq.in. nearly.}$$

Since the mean radius of cone is 8 ins. the width of cone is

$$W = \frac{S}{2\pi R} = \frac{100}{2\pi 8} = 2 \text{ ins. (approx.).}$$

The total axial pressure required to throw the clutch into engagement is

$$P_a = \frac{P_t \sin \alpha}{\mu}$$

$$= 395\left(\frac{.208}{.2}\right) = 410 \text{ lbs.}$$

In the *disc clutch* the friction surfaces are planes perpendicular to the axis of the shaft. If more than two discs are used the clutch is termed a multiple disc clutch.

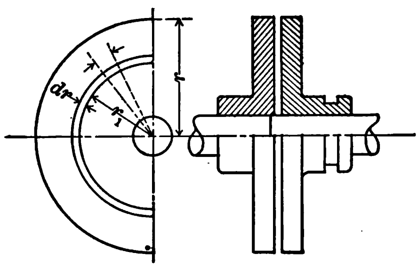


FIG. 7–10.

In Fig. 7–10 is shown a disc clutch in a very elementary form. One disc is rigidly keyed to its shaft while the other is on a feather key and may be pressed against the first by an axial force. Then if

p = normal pressure per square inch between discs,
r = outside radius of discs,
M_f = moment of the friction about center of shaft in inch-pounds,
T = torque to be transmitted by clutch,
P_a = total axial pressure.

If the disc is assumed to extend to the center of shaft, then

$M_f = \frac{2}{3}\mu P_a r$,*

but M_f must be at least equal to the torque to be transmitted, or

$$T \overline{\overline{<}} \tfrac{2}{3}\mu P_a r. \qquad (4)$$

Where much power is to be transmitted this would give excessive axial pressures; to avoid this the multiple disc clutch is used.

* See Appendix A for development of this equation.

Here the friction surfaces consist of two sets of rings as shown in Fig. 7–10*a*. One set is notched at outer circumference, these notches fitting over an equal number of projections on outer casing, thus preventing relative rotation. The other set is notched at

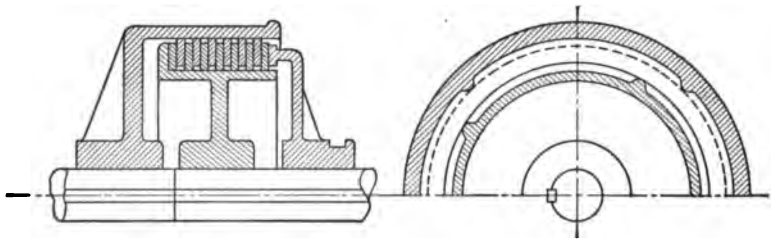


FIG. 7–10*a*.

inner circumference and a corresponding number of notches on inside drum of clutch fit into these notches, the discs being assembled alternately one of each set.

Let R = outside radius of discs,
r = inside radius of discs,
n = number of pairs of surfaces in contact.

Then approximately

$$T = P_a \mu n \frac{R+r}{2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$P_a = \frac{2T}{\mu n (R+r)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Thus by increasing the number of pairs of friction surfaces, n, we can decrease the axial thrust P_a. This type of clutch is much used in automobiles and hoisting machinery.

In the **cylindrical clutch** (Fig. 7–11) the friction surfaces are cylinders, or portions of cylinders, concentric with the shaft. In this clutch if R is the radius of friction surface, P_n is the total normal pressure between surfaces, then

$$T \lesseqgtr \mu P_n R.$$

The methods of applying the normal pressure are various, generally by means of levers, toggles, or screws. This type of clutch is very generally used in shops and for power transmission.

The **coil clutch** (Fig. 7–12) consists of a steel coil wound on a chilled cast-iron drum. One end of this coil is fastened to a sleeve or hub which is keyed to the shaft to be put in motion. By

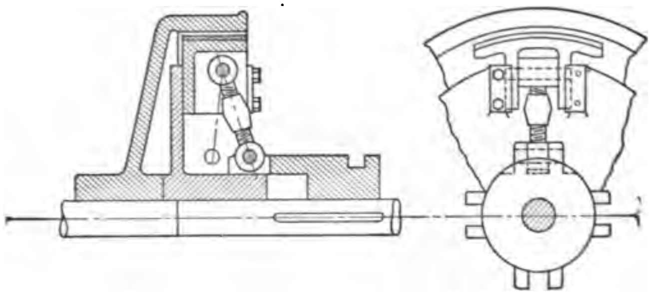


FIG. 7–11.

means of a sliding sleeve and a lever a pull is exerted on the free end of coil. This causes it to tighten around the drum, each turn increasing the friction between drum and coil. In Fig. 12–13 is shown a small portion of the coil. The force required

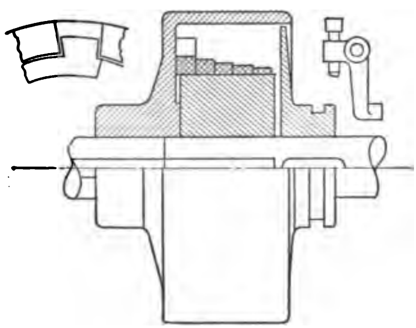


FIG. 7–12.

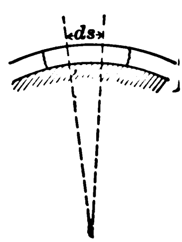


FIG. 7–13.

to make this slip to the left is the force tending to move it to the right plus the friction between this portion of coil and the drum. Thus the force P which must be exerted to make the band or coil slip over drum is at least

$$P = Q + \text{total friction.}$$

Let μ be the coefficient of friction, α the total angle in radian measure, subtended by coil, and e the base of the Napierian

logarithms $=2.718$, then the relation between P and Q is given by the equation

$$\frac{P}{Q}=e^{\mu\alpha}.$$

In this equation P represents the pull that the clutch can transmit if a pull of Q lbs. is exerted at loose end of coil. For a lubricated steel band on a cast-iron drum μ may be taken at about $\frac{1}{10\pi}$ then

for a coil of 1 turn $P=e^{1/5}Q=1.22\ Q$,
for a coil of 2 turn $P=e^{2/5}Q=1.5\ Q$,
for a coil of 4 turn $P=e^{4/5}Q=2.2\ Q$,
for a coil of 8 turn $P=e^{8/5}Q=5\ Q$.

PROBLEMS

1. Determine the stress in a sleeve coupling for a 2½-in. shaft, the stress in shaft being 10,000 lbs. per square inch. Take dimensions of coupling according to empirical equations in text.

2. Two 4-in. shafts are coupled by means of a flange coupling. Find diameter of bolts if there are six of them and the stress in bolts may be 6000 lbs. per square inch, while that in shaft is 9000 lbs. per square inch.

3. Design and make sketch of a flange coupling for a hollow shaft 12 ins. outside and 6 ins. internal diameter.

4. What horse-power can be transmitted by a cone clutch the mean radius of which is 8 ins.? The width of friction surfaces is 4 ins. and the maximum normal pressure on same is 20 lbs. per square inch. The angle of cone is 10° and assume $\mu=.12$. Shaft runs at 100 R.P.M.

5. Design and sketch a multiple disc clutch to transmit 20 H.P. at 300 R.P.M. The outside diameter not to exceed 14 ins. The discs to be alternately steel and bronze. State all necessary assumptions.

6. In a cylindrical clutch the diameter of the friction surface is 18 ins. and the breadth is 4 ins. The clutch shoes subtend an angle of 180° on clutch rim. The pressure per square inch is 35 lbs. What horse-power can be transmitted at 100 R.P.M. if coefficient of friction is .15?

CHAPTER VIII

JOURNALS AND BEARINGS

Journals. That part of a rotating spindle, shaft, or axle, which rests in a support is called a journal, while the support itself is the bearing. As the journal is part of the shaft the calculation of its diameter for strength is a part of the design of the shaft, and this was considered in Chapter VI. It is rarely, however, that the strength of the journal alone is the important consideration. The fact that there is friction between a journal and its bearings and that therefore heat is generated is largely the determining factor in calculating the journal dimensions. The bearing must have sufficient area to dissipate the heat generated into the surrounding atmosphere, otherwise the temperature of the bearing would soon rise to a point where the lubricant loses its lubricating qualities, causing destruction of both journal and bearing.

Bearing Pressure. By the area of a journal is meant the projected area, that is, the cylindrical area of the contact surfaces projected on a plane parallel to the axis of the shaft. Thus, if d is the diameter of a journal and l is its length both in inches the projected area is dl. If W is the total load on the journal then the bearing pressure per square inch of projected area is

$$p=\frac{W}{ld}. \qquad (1)$$

The safe value of p depends on the speed of rotation, on the materials used for journal and bearing, on the quality of lubricant and method of lubrication, and on the type of machine. Table 1 gives values of p, for different kinds of bearings, as commonly found in practice.

TABLE 1

Kind of Bearing.	$\frac{p}{\text{Lbs.}}$
Bearings for slow speeds and intermittent loads	2000–4000
Main journals of steam engines	150– 300
Crank-pins of high-speed engines	250– 600
Crank-pins of low-speed engines	700–1500
Cross-head pins for high-speed engines	800–1600
Cross-head pins for low-speed engines	1000–2000
Motor and generator bearings	30– 80
Line shaft bearings	

Heating of Journals. The amount of heat generated by the friction of a journal depends on the load, the rubbing speed and the coefficient of friction of the lubricant and surfaces. As the rubbing speed increases with the diameter it is essential, wherever possible, that this be kept as small as considerations of strength will permit. The proper length of a journal is determined by its liability to heating.

Let d = diameter of journal in inches,
l = length of journal in inches,
N = revolutions per minute,
μ = coefficient of friction,
W = total load on journal;

then the work of friction is

$$A=\frac{\pi dNW\mu}{12} \text{ ft.-lbs.}; \qquad (2)$$

and the work per square inch of projected area is

$$a=\frac{A}{ld}=\frac{\pi NW\mu}{12l}; \qquad (3)$$

therefore.

$$l=\frac{\pi NW\mu}{12a}. \qquad (4)$$

The amount of heat generated per square inch of projected area in British thermal units is the work of friction divided by 778 or

$$q=\frac{a}{778}=\frac{\pi\mu NW}{778\times 12l}. \qquad (5)$$

With well constructed and properly lubricated bearings a value of .02 may be assumed for μ; this gives

$$q = .0000067\frac{NW}{l}. \qquad \qquad (6)$$

The value of q varies from .2 to 1.0 or

$$l = 0.000034NW \text{ to } 0.0000067NW.$$

The ratio of length, l, of journal to diameter, d, as usually found in practice is given in Table 2. In calculating the length of a bearing it is best to assume this ratio from the table and check up for p and q from equations 1 and 6.

TABLE 2

Kind of Bearing.	$\frac{l}{d}$
Transmission bearings:	
Line shaft, countershaft, etc.	3–5
Heavy pillow blocks	2½–3½
Main journals of steam engines	1¾–2½
Cross-head pins	1—2
Crank-pins	¾–1½

EXAMPLE. A shaft carries a gear weighing 500 lbs. at its end as in Fig. 8-1. The pressure on gear teeth acts vertically downward and is 300 lbs. The twisting moment to be transmitted is 6000 in.-lbs. The speed is 200 R.P.M. Design a suitable bearing (pillow block). The bending moment is

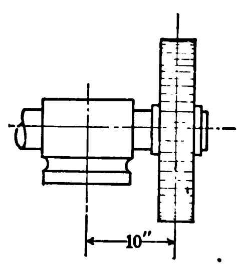


FIG. 8-1.

$$M = (300+500)10 = 8000 \text{ in.-lbs.}$$

The ideal twisting moment is

$$T_1 = 8000 + \sqrt{6000^2 + 8000^2} = 18{,}000 \text{ in.-lbs.};$$

then the diameter of shaft and journal is

$$d = 1.72\sqrt[3]{\frac{18000}{9000}} = 2.16, \text{ say } 2\tfrac{1}{4} \text{ ins.}$$

Assume length of pillow block is $3d$ or

$$l = 3 \times 2\tfrac{1}{4} = 6\tfrac{3}{4} \text{ in.}$$

The projected area of journal is

$$s = 2\tfrac{1}{4} \times 6\tfrac{3}{4} = 15.19 \text{ sq.in.};$$

and pressure per square inch of projected area is

$$p = \frac{800}{15.19} = 53, \text{ which is a safe value.}$$

The heat generated per square inch of projected area is

$$q = .000067 \frac{NW}{l} = .16;$$

this is a low value and therefore satisfactory.

Bearings. There are two general types of bearings, viz.: *journal* bearings and *thrust* bearings. A journal bearing is one

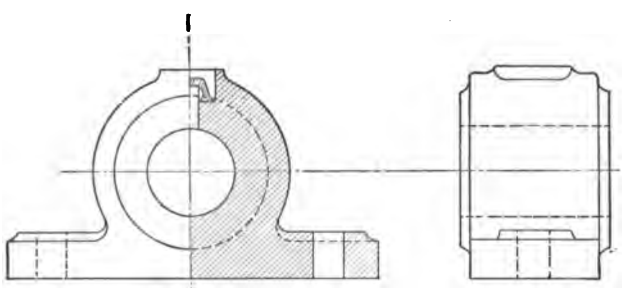


FIG. 8-2.

which supports a load acting perpendicular to axis of shaft. In thrust bearings the load acts parallel to shaft axis. A bearing sometimes combines these functions.

Steps or Brasses. Except for very low speeds or very light pressure, bearings are always lined with a suitable lining called steps, boxes, or brasses. These steps are usually a white metal alloy such as babbitt, a brass or a bronze alloy. There is a wide variation of bearing metal alloys on the market and some diversity of opinion as to their respective merits. For light and medium pressures the white metal alloys are preferred and are commonly

used for transmission bearings and the main journal bearings steam engines. For medium and heavy pressures and also for high speeds brass or bronze linings are used, the latter being much the better of the two. They are also used where minimum wear and accurate adjustment are important, as in machine tools. For very high pressures, particularly in thrust bearings, hardened steel on hardened steel is the best combination. The thickness of white metal linings may be

$$t=\frac{\sqrt{d}}{6}.$$

For brass or bronze steps

$$t=\tfrac{3}{40}d+\tfrac{1}{8}\text{ in.}$$

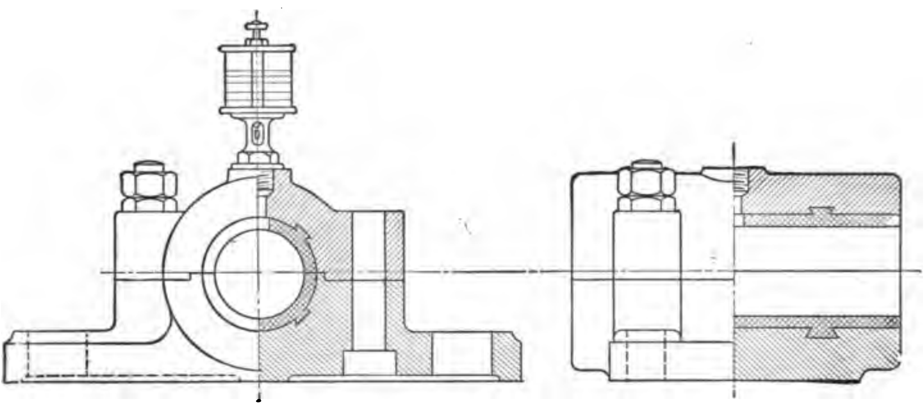


FIG. 8-3.

Frequently the steps are of cast iron or brass with a white metal lining. Figs. 8-5 and 8-6 shows some of the usual types of steps.

Lubrication. To prevent destruction of journal or bearing it is essential to lubricate them. Perfect lubrication consists in maintaining an unbroken film of the lubricant between the journal and its bearing, so that there will not be a metal to metal contact. The lubricant is introduced at the point of least pressure, which in a journal bearing is usually on top. The simplest method of lubrication is to cast an oil well in cap (Fig. 8-2) and feed the lubricant to journal by means of a wick. A better way is to use a drip oil or grease cup (Fig. 8-3); this permits of regulating the quantity of lubricant fed. In the ring oiling bearing (Fig.

8–8) a continuous supply of oil is provided by means of one or more rings (or chains) which dip into a reservoir of oil, and due to their rotation with shaft, carry the oil to top of journal. Probably the best way of all is forced lubrication. In this system

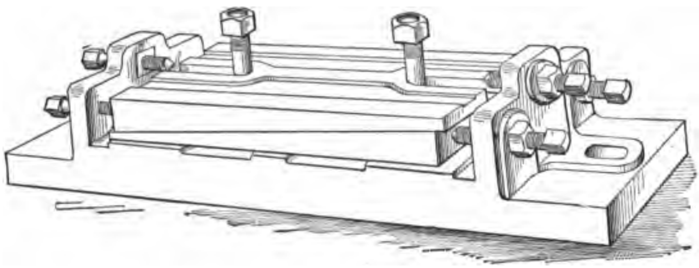


FIG. 8–4.

a raised storage oil tank or a small pump forces the requisite amount of lubricant through small tubes to the various bearings of the machine. This system is largely used in modern steam and gas engine practice as well as for motor cars.

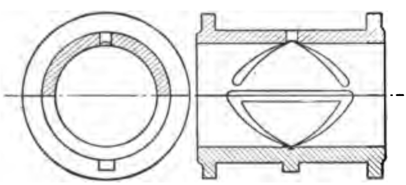


FIG. 8–5.

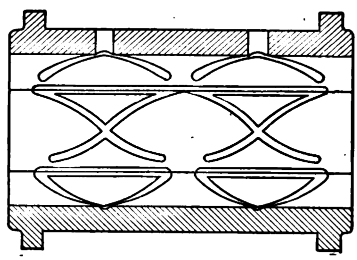


FIG. 8–6.

Oil Grooves. To provide proper distribution of the lubricant the steps are provided with oil grooves. The forms of these are shown in Figs. 8–5 and 8–6. Their dimensions range from a width of $\frac{3}{16}$ in. and depth of $\frac{1}{8}$ in., for shafts 2 to 3 ins. diameter, to a width of $\frac{1}{2}$ in. and depth of $\frac{5}{16}$ in. for shafts of 18 to 20 ins. diameter. Fig. 8–5 represents the brasses of a two-part bearing while Fig. 8–6 shows those of a four-part bearing.

Adjustment of Bearings. To provide an adjustment for journal bearings when they are worn, they are divided into two or more parts. The top part is called the cap. The shaft hanger shown in Fig. 8–6 has the hearing divided horizontally into two parts.

The post bearing (Fig. 8–7) is divided on a 45° line. Crankshaft journal bearings are often divided into four parts, thus giving both up and down, and a sidewise adjustment.

It is also frequently required to give the bearing a slight motion to enable it to be accurately lined up with other bearings. This may be done by supporting the bearings by screws as in the shaft hanger (Fig. 8–8), or by placing it on a sole plate with wedge adjustment as in Fig. 8–4.

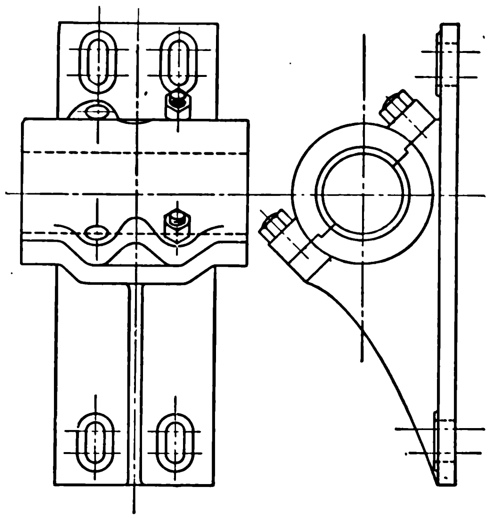


FIG. 8–7.

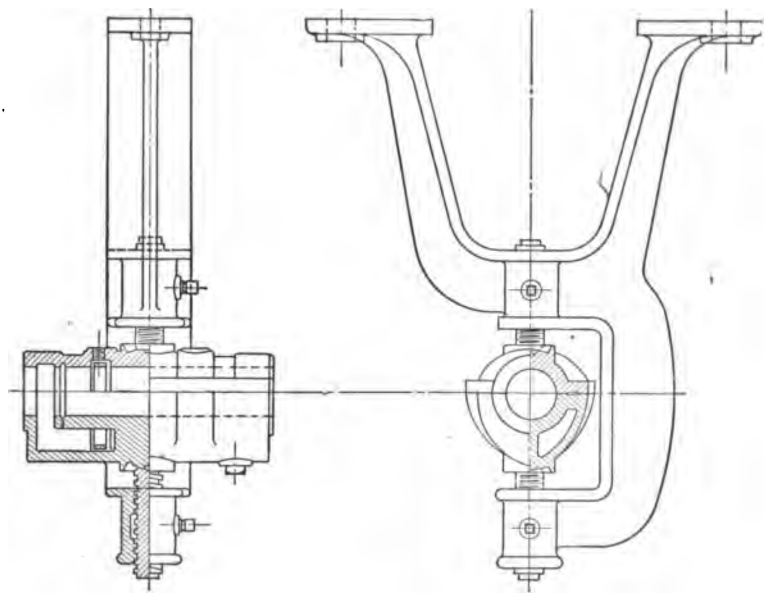


FIG. 8–8.

Thrust Bearings. There are two types of thrust bearings, viz.: pivot bearings and collar bearings. In the pivot bearing

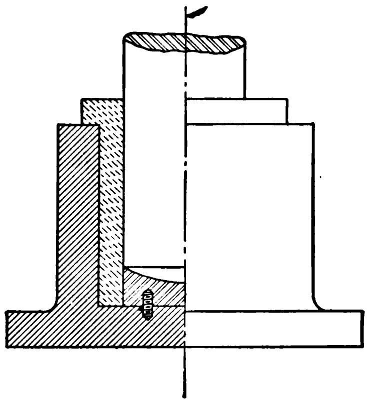


Fig. 8–9.

the end of the shaft is the bearing surface, while in the collar bearing one or more collars are formed on shaft and the thrust

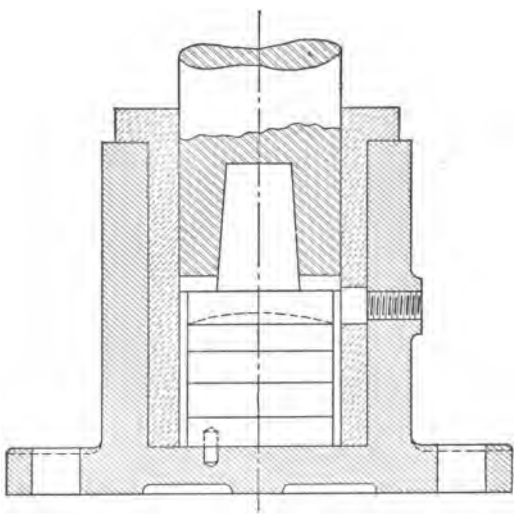


Fig. 8–10.

is taken by them. Fig. 8–9 illustrates the simplest type of pivot bearing in which the end of the shaft rests on a hardened steel

washer. Fig. 8–10 shows a better form of bearing in which a separate end piece of hardened steel is let into the shaft and a number of washers are used; these are alternately steel and brass.

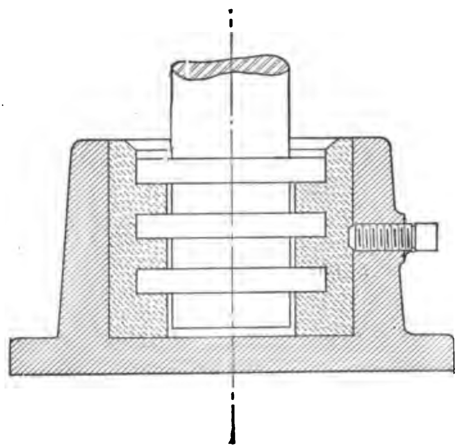


FIG. 8–11.

Collar bearings are used chiefly on horizontal shafts which have to sustain a large axial thrust, as the propeller shafts of

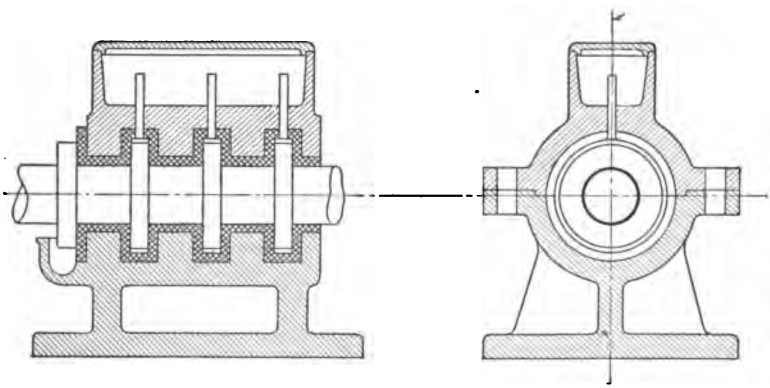


FIG. 8–12.

steamboats. Fig. 8–11 shows a collar bearing for a vertical shaft and Fig. 8–12 one for a horizontal propeller shaft for a small steamer.

Friction Work in Pivot and Collar Bearings. It may be assumed that the pressure is distributed uniformly over the surface. Then for pivot bearings:

Let p = pressure in pounds per square inch,
R = radius of bearing surface in inches,
W = total load in pounds,
N = revolutions per minute,
M_f = moment of friction in inch-pounds,
μ = coefficient of friction;

then

$$p=\frac{W}{\pi R^2};$$

and

$$M_f=\tfrac{2}{3}\mu RW. \quad \text{(See Friction disc clutch.)}$$

The work of friction is

$$A=2\pi N\tfrac{2}{3}\frac{\mu RW}{12}=.35\mu RNW \text{ ft.-lbs.} \quad . \quad . \quad . \quad (7)$$

For collar bearings or pivot bearings in which the contact surface is a ring,

Let R = outside radius in inches,
r = inside radius in inches,
n = numbers of collars;

then

$$p=\frac{W}{\pi(R^2-r^2)n},$$

$$M_f=\tfrac{2}{3}\frac{R^3-r^3}{R^2-r^2}W\mu.$$

$$\therefore \quad A=\frac{2\pi NM_f}{12}=.35\mu NW\frac{R^3-r^3}{R^2-r^2}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From these equations it is evident that the work lost in friction increases with the radius R and it is therefore advantageous to keep the radius as small as possible. In collar bearings the diameter of collars may be from $1.3d$ to $1.6d$, where d is the diameter of shaft. Table 3 gives safe values of p for pivot and collar bearings. These values must be taken as very approximate as information on this subject is widely varying.

TABLE 3

	Lbs.
Very low speeds:	
Column cranes, turn-tables, etc.	1500–2000
Turbine foot-step bearings:	
Very high grade construction	700–1200
Collar bearings:	
Propeller shafts	60–90

EXAMPLE. The thrust bearing of a steam yacht is to be designed. The engine is 500 I.H.P. and the propeller 9 ft. pitch, the shaft being $5\frac{1}{2}$ ins. diameter. The engine makes 220 R.P.M.

If we assume a slip of 20 per cent the boat will be driven 7.2 ft. for each revolution of propeller or 1580 ft. per minute. But the pressure on thrust bearing times the speed in feet per minute is the work done per minute in foot-pounds. In designing thrust bearing this is equated to the I.H.P. of the engine. Then

$$1580 \times W = 500 \times 33{,}000,$$

$$W = 10{,}450 \text{ lbs.}$$

The outside diameter of collars may be 8 ins., and p we will assume to be 60 lbs. Then each collar can support a thrust

$$W' = \frac{\pi}{4}(8^2 - \overline{5.5}^2)60 = 1600 \text{ lbs. nearly};$$

and therefore the number of collars required is

$$n = \frac{10450}{1600} = 6.55, \text{ say 7 collars.}$$

The work lost in friction is

$$A = .35\mu 220 \times 10450 \frac{\overline{4}^3 - \overline{2\tfrac{3}{4}}^3}{\overline{4}^2 - \overline{2\tfrac{3}{4}}^2};$$

and if bearing is lubricated in oil bath μ may be assumed at .01, then

$$A = 41300 \text{ ft.-lbs.}$$

$$= 1\tfrac{1}{4} \text{ H.P.}$$

Ball Bearings. The heavy friction losses occasioned by ordinary journal bearings has led to the development of ball-and-roller bearings. In these types rolling friction is substituted for sliding friction. Ball bearings were first applied extensively in the bicycle. Until the publication of Prof. Stribeck's research work it was commonly held that ball bearings were suitable only for very light loads. They are now used in great variety of machines under all conditions of load and speed.

Radial ball bearings are those supporting loads at right angles to the axis of the shaft. Fig. 8–13 shows a radial bearing consisting of a single row of balls placed between an inner and an outer ring called races. The races may be straight or grooved, the radius of the groove being about two-thirds the ball diameter. In order to assemble this bearing it is necessary to cut a notch at right angles to the groove. In these bearings the balls are in contact with each other and since the points of contact move in opposite direction there is considerable friction and wear. To obviate this the balls are separated by a light metal cage as shown in Fig. 8–14, this being now the almost universal custom for medium and heavy-load ball bearings.

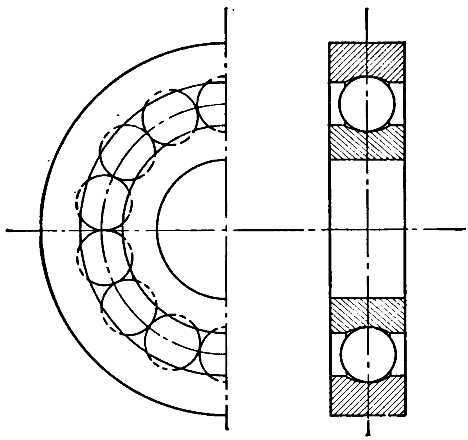


FIG. 8–13.

Allowable Loads. Radial Bearings. The total load which a radial ball bearing may safely carry depends on the number of balls and their diameter.

Let P = total load,
p = load on one ball,
n = number of balls in bearing,
d = diameter of balls in $\frac{1}{8}$ in.,
k = a constant.

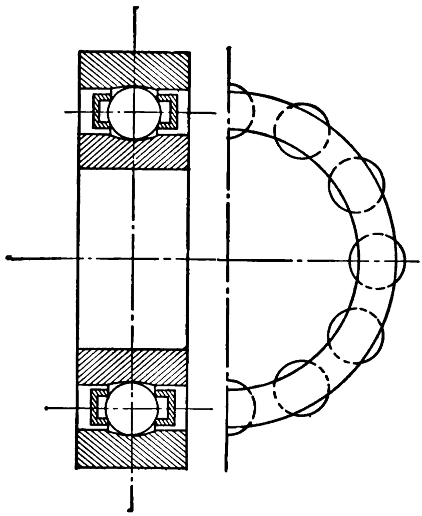


FIG. 8-14.

The load may be considered as being carried by one-fifth the number of balls in bearing or

$$p = \frac{5P}{n}. \quad \ldots \ldots \ldots \quad (9)$$

The safe load for one ball is

$$p = kd^2; \quad \ldots \ldots \ldots \quad (10)$$

the value of k may be taken from 10 to 15 for first-class material and grooved races, and about one-half of these figures for straight races. The above equations do not consider speed of rotation, and this would not affect the carrying capacity if it were absolutely uniform and the load steady. Such conditions, however, are rarely met with in practice, and therefore with increasing speeds and varying loads the capacity should be decreased.

In **angular bearings** the constraining surfaces are at an angle to the axis of the shaft. Thus Figs. 8-15, 8-16, 8-17, show re-

spectively two-, three- and four-point contact bearings. In order to insure true rolling of the balls it is essential that the lines through

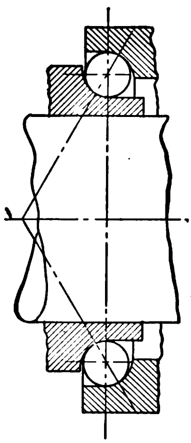


FIG. 8-15.

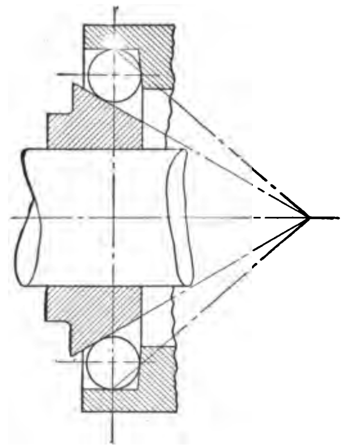


FIG. 8-16.

the points of contact intersect in a point on the axis of the shaft or that they be parallel to this axis.

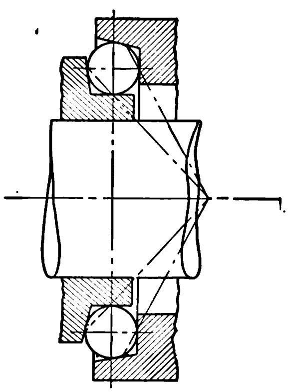


FIG. 8-17.

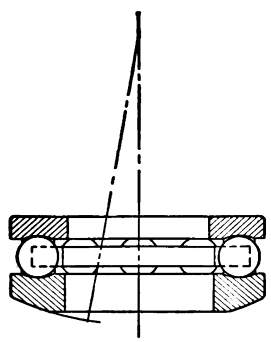


FIG. 8-18.

The importance of high-grade material and workmanship can hardly be overestimated. The material for both balls and races is a special grade of cast steel. This must be of uniform

hardness and structure. The balls are highly polished so that no grinding scratches can be detected. In the best bearings their diameters do not vary by more than .0001 in.

Experiments have shown that the coefficient of friction is almost independent of the speed of rotation. Its mean value may be taken at .0015 for a normally loaded bearing.

Ball Thrust Bearings. These are used for all loads and speeds. For very low speeds, as in the steering gear of motor cars, turn

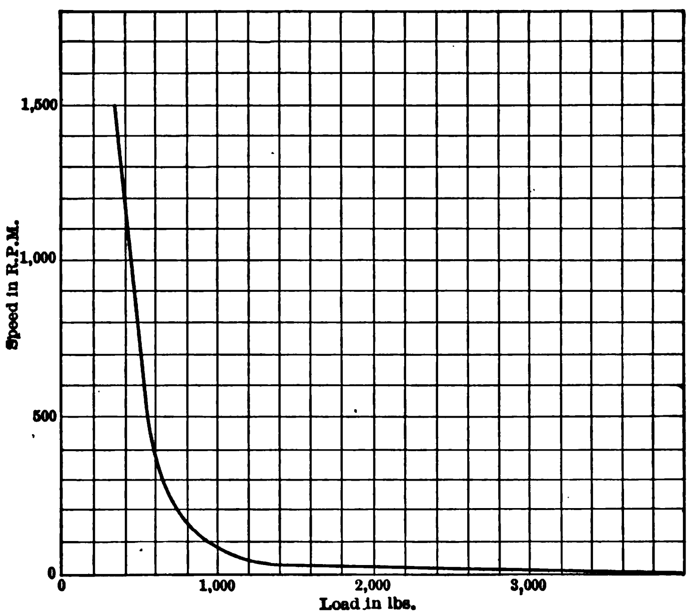


FIG. 8–19.

tables, cranes, etc., the races are filled with balls, for higher speeds cages are used as shown in Fig. 8–18. In thrust bearings the total load is of course

$$P = np. \quad \ldots \ldots \ldots \quad (11)$$

This assumes a uniform distribution of the load on all the balls. To insure this it is usual to use a spherical seat for the thrust washer supported by the fixed frame. In thrust bearings the safe load per ball decreases rapidly with increasing speeds. Thus in the equation $p = kd^2$ the value of k is no longer constant but

depends on the speed of shaft. Just what the variation in k should be is not definitely known and we can only depend upon the accumulated experience of the manufacturers of such bearings.

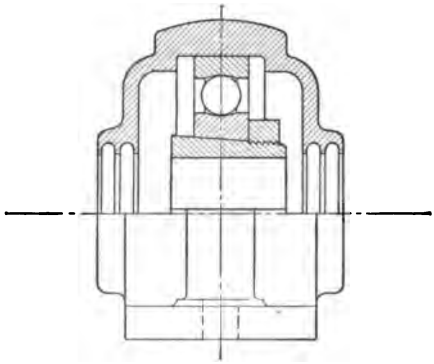


FIG. 8–20.

Fig. 8–19 shows safe loads for speeds from 10 to 1500 revolutions per minute for a thrust bearing containing eighteen $\frac{1}{4}$-in. balls, as recommended by a well-known manufacturer. This shows values of k from 4.6 to 33.4.

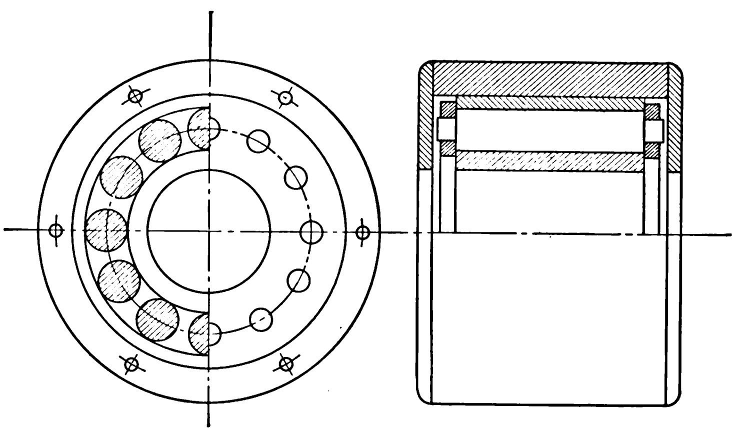


FIG. 8–21.

Cleanliness is essential to the success of all forms of ball bearings. They must be thoroughly protected from grit and dirt. If a shaft does not pass through the bearing the outer end is closed

by a dust-proof cover. Where the shaft passes through the bearing frame should be bored out $\frac{1}{64}$ to $\frac{1}{32}$ in. larger than shaft, and a groove turned in this bore as shown in Fig. 8–20. A small hole is drilled on the inside to communicate with the oil space; the edges of this groove must be sharp. Sometimes a felt washer is used to make a dirt-tight closure. The bearings

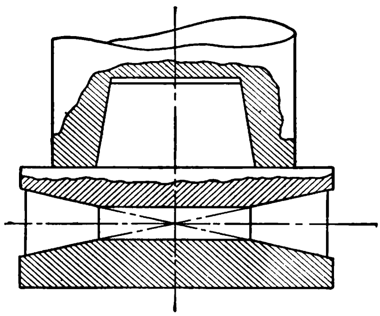


FIG. 8–22.

should be lubricated with a good quality of oil, free from acid which will not attack the highly polished surfaces of balls or races.

Roller Bearings. In the simplest type of roller bearing the space between shaft and housing is filled with rollers. There is in this type of bearing, however, a tendency for the rollers to get out of alignment, which causes rapid destruction of the bearing. For this reason the rollers are more usually carried in a suitable

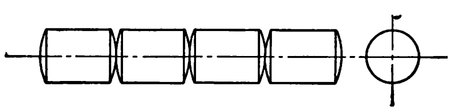


FIG. 8–23.

cage as shown in Fig. 8–21. In the Hyatt roller bearing the rollers are made of a strip of steel rolled into a cylinder. This gives sufficient flexibility to the roller to insure contact along the entire length of same, resulting in a uniform distribution of the load on it as well as the surface on which it rolls. Roller bearings have also been used as thrust bearings. The rollers may be conical (Fig. 8–22) or if cylindrical divided into a number of short sections (Fig. 8–23).

Design. The safe load on a roller depends on its diameter and length.

Let d = roller diameter in inches,
l = roller length in inches,
n = number of rollers in bearing,
v = velocity of roller in feet per minute,
P = total safe load;

then

$$P = knld^2.$$

The value of k depends on the velocity v; the following table gives average practice:

v(feet per minute) =	0 to 20	100	200	300	400	500
k =	200	50	22	17	13	10

PROBLEMS

1. The main journals of a steam engine are 6 ins. diameter by 10 ins. in length. The load supported on each bearing is 14,000 lbs. The speed of engine is 120 R.P.M. Find horse-power lost in friction at each bearing if $\mu = .04$.

2. A shaft, supported on two bearings 7 ft. between centers, carries a gear located 2 ft. from one of the bearings. The load on shaft due to weight of gear plus tooth pressure is 22,000 lbs. Design the journals so that the pressure per square inch of projected area is 180 lbs. and assume ratio $l:d = 2\frac{1}{2}$.

3. In a steamboat the total thrust on the propeller thrust bearing is 50,000 lbs. The engine develops 3000 H.P. at 180 R.P.M. The propeller shaft is hollow with the hole $\frac{2}{3}$ of the outside diameter. Find diameter of shaft, allowing a stress of 8000 lbs. per square inch. Determine the number of thrust collars required if their outside diameter is $1\frac{1}{2}$ shaft diameter and the bearing pressure is 75 lbs. per square inch.

4. Design the foot-step bearing for a column crane. The maximum load on crane is 20 tons and the weight of crane is 8 tons.

CHAPTER IX

BELTS AND PULLEYS

Belts are made of various materials, such as leather, cotton, hemp and rubber. Leather is by far the commonest and best material, oak-tanned ox-hide being used for the highest grade belts. The strongest part of the hide is a strip 12 to 16 ins. wide and about 4½ ft. in length, along the back of the animal. This strip has the most uniform elastic qualities, its thickness is from .20 to .25 in. For narrow and inferior belts the side of the hide is used; the thickness of this part of the hide being from .25 to .35 in. In manufacturing a belt these strips are beveled off at the ends, glued, sewn or riveted together until the desired length is obtained. Belts may be single, double or triple, according to there being one, two or three thicknesses of leather.

Cotton, hemp and rubber belts are used chiefly in exposed or damp locations. They are cheaper than leather belts, but do not last as long. Cotton belts are either woven endless or made by sewing together 4 to 10 thicknesses (plies) of canvas or duck. Rubber belts are made by cementing several plies of canvas with a rubber composition.

Belt Fasteners. There are many different methods of joining the ends of leather belts. The best method, and the one usually adopted for large belts and important drives, is to scarf the ends and cement them together. This makes the joint nearly as strong as the rest of the belt. In small belts the joint is generally made by lacing the ends together with rawhide or wire. This joint is only from one-half to two-thirds as strong as the belt. It readily permits of taking up the stretch in new belts and as this amounts to about 6 per cent of its length, it has to be done more or less frequently in long belts in order to maintain the proper driving tension. There are a number of patented metallic belt fasteners on the market, some of which are shown in the belt joints (Fig. 9–15).

Belt Transmission. Fig. 9–1 represents two pulleys connected by a belt. It is evident that in order to produce rotation of the follower the moment of the friction between belt and pulley about center B must be at least equal to the moment of the resistance to be overcome. This friction is due to the initial tension put on the belt when it is placed on the pulleys, or, in order words, the

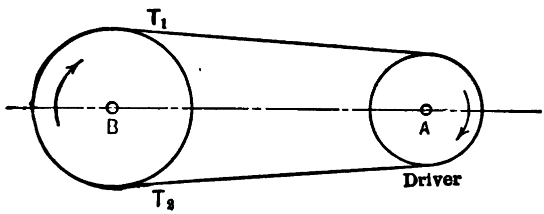


FIG. 9–1.

tension in belt when at rest. Let T_3 be this tension. When the driver starts to move it pulls on the driving or tight side of belt, increasing the tension there and decreasing that of the opposite or slack side until the difference between these tensions is sufficient to overcome the resistance to motion offered by the follower. Then if

T_1 = tension in tight side,
T_2 = tension in slack side,
P = driving force exerted at rim of pulley;

we have

$$T_3 = \frac{T_1 + T_2}{2}, \quad \dots\dots\dots \quad (1)$$

$$P = T_1 - T_2. \quad \dots\dots\dots \quad (2)$$

The difference of T_1 and T_2 is equal to the friction of the belt on the pulley rim. The relation between them being given by the equation

$$\frac{T_1}{T_2} = e^{\mu\theta}, \quad \dots\dots\dots \quad (3)^*$$

where e is the base of the natural logarithms ($e = 2.718$), θ is the angle of contact of belt and pulley rim expressed in radian measure ($180° = \pi$ radians) and μ is the coefficient of friction.

* See appendix B for derivation.

Since T_1 is the maximum tension in belt it will determine the necessary cross-sectional area and therefore the width required; and if

A = sectional area of belt in square inches,
s_t = safe tensile stress in pounds per square inch;

then

$$T_1 = As_t, \quad \quad (4)$$

or if the safe tension per inch of width be designated by k and width of belt by b, then

$$T_1 = bk. \quad \quad (5)$$

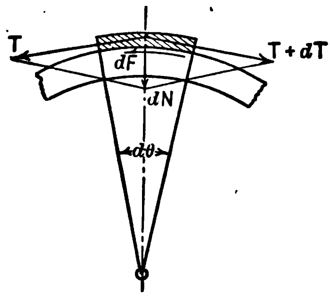


FIG. 9–2.

The following table gives values of $e^{\mu\theta}$ for various values of μ and θ:

μ	Portion of Circumference in Contact.							
	.2	.3	.4	.45	.50	.55	.6	.7
.25	1.37	1.60	1.87	2.03	2.19	2.37	2.57	3.00
.28	1.42	1.69	2.02	2.21	2.41	2.63	2.81	3.43
.33	1.51	1.86	2.29	2.54	2.82	3.13	3.47	4.27
.38	1.61	2.05	2.60	2.93	3.30	3.72	4.19	5.32
.40	1.65	2.13	2.73	3.10	3.51	3.98	4.52	5.81
.50	1.87	2.57	3.51	4.11	4.81	5.63	6.59	9.00

The angle θ should be the smaller angle subtended by a belt on a pair of pulleys. For an open belt this will be $\theta° = 180° - 2\left(\sin^{-1}\frac{C}{R-r}\right)$ where R and r are the radii of the two pulleys and C is the distance between centers.

In the above equations the effect of centrifugal force has been neglected. For belt speeds above 2500 ft. per minute this force should be taken into account. Its effect is to increase the tension in the belt without increasing its driving power and therefore the safe value of T_1 should be reduced by an amount equal to the centrifugal force.

Let w = weight of 1 cu.in. of leather,
R = radius of pulley in inches,
g = acceleration of gravity in feet per second,
c = centrifugal force of 1 in. length of belt,
v = velocity of belt in feet per second;

then

$$c = \frac{w}{g} \frac{Av^2}{\frac{R}{12}}.$$

The sum of all these forces (Fig. 9-3) in a direction parallel to the belt is

$$C = 2Rc = \frac{24Av^2w}{g},$$

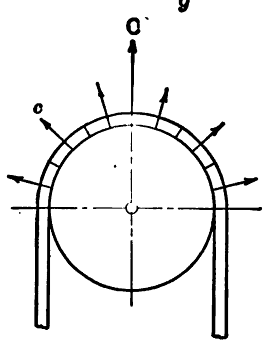


FIG. 9-3.

and therefore the tension due to centrifugal force produced in belt per square inch of cross-sectional area is

$$c_t = \frac{C}{2A} = \frac{12wv^2}{g}.$$

The average weight of leather may be taken at $w = .035$ lb., $g = 32.2$;
then

$$c_t = .013v^2;$$

and therefore

$$T_1 = A(s_t - .013v^2); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

from which required width of belt may be calculated.

The value of the coefficient of friction, μ, varies between rather wide limits, depending on the material of the belt and of the pulleys, on the amount of slip of the belt, and to some extent on its speed. An average value for a leather belt on a cast-iron pulley is .33. For pulleys made of pulp or wood it is somewhat less, except when new, and for paper pulleys considerably more.

The breaking strength of good leather belts varies from 3500 to 4500 lbs. A safe stress of 300 lbs. per square inch for belts in which the ends are cemented together is good practice, although the smaller this stress the longer the life of the belt.

EXAMPLE. A 15-H.P. motor drives a line shaft. The motor makes 600 R.P.M. and has a 12-in. pulley on armature shaft. Find width of double belt required if .4 of pulley circumference is in contact with belt.

The belt velocity is

$$v = \frac{\pi \times 12 \times 600}{12} = 1890 \text{ feet per minute};$$

therefore the effect of centrifugal force may be neglected. The tangential force at rim of pulley is

$$P = \frac{15 \times 33000}{1890} = 262 \text{ lbs.}$$

If μ be assumed at .33 then from the table

$$\frac{T_1}{T_2} = e^{\mu\theta} = 2.29,$$

then

$$T_1 - T_2 = P = 262,$$

$$T_1 - \frac{T_1}{2.29} = 262;$$

$$\therefore \quad T_1 = 455 \text{ lbs.}$$

This is the maximum tension which occurs in belt. The thickness of a double belt is about $\frac{3}{8}''$. Allowing a tension of 300 lbs. per square inch the cross-sectional area required is

$$A = \frac{455}{300} = 1.52 \text{ sq.in.}$$

and therefore the width is

$$b=\frac{1.52}{.375}=4.06\text{-ins., or say 4-in. belt.}$$

Practical Methods. It is not always possible to predetermine the exact conditions under which a belt is to run, and various practical rules are in use for determining the width of a belt to transmit a given horse-power. In general it will be found that $\frac{T_1}{T_2}=2.5$ gives satisfactory results; then

$$T_1=\tfrac{5}{3}P.$$

If b is the width of belt in inches and k is the allowable tension in belt per inch of width then

$$b=\frac{T_1}{k}=\frac{5P}{3k}, \qquad \text{(7)}$$

but

$$P=\frac{\text{H.P.}\times 33000}{v};$$

and

$$\therefore \quad b=\frac{55000\times\text{H.P.}}{vk}. \qquad \text{(8)}$$

The thickness, t, of belts, the effective pull, P, per inch of width and the corresponding maximum tension, k, per inch of width, on tight side of belt, as found in American practice, is given in table below. In obtaining k it was assumed that $\frac{T_1}{T_2}=2.5$.

	t	P	k
Single belt	$\frac{5}{32}$–$\frac{8}{32}$	25–40	42– 67
Double belt	$\frac{9}{32}$–$\frac{14}{32}$	35–60	58–100
Triple belt	$\frac{15}{32}$–$\frac{22}{32}$	55–80	92–142

A rough rule of thumb is to allow 65 sq.ft. of belt to pass over pulley for each horse-power transmitted, or, if A is belt area, passing over pulley, in square feet per minute, we have

$$A=\frac{bv}{12},$$

$$\text{H.P.}=\frac{A}{65}=\frac{bv}{780};$$

and

$$b=\frac{780\times\text{H.P.}}{v}\text{ or say }\frac{800\times\text{H.P.}}{v}. \quad . \quad . \quad . \quad (9)$$

For a double belt 40 sq.ft. may be allowed per horse-power or,

$$\text{H.P.}=\frac{A}{40}=\frac{bv}{480};$$

and

$$b=\frac{480\text{ H.P.}}{v}\text{ or say }\frac{500\times\text{H.P.}}{v}. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

General Rules for Installation. Belts may be used to connect two pulleys wherever a small variation of angular velocity ratio is of no consequence. For best results there are certain limitations as to center distance between shafts. If the distance is very short a tight belt is required, which shortens its life and increases the wear on shaft bearings. One rule is to make the minimum distance between centers twice the diameter of the larger pulley. Belts are rarely used for center distances of more than 60 ft., as it is cheaper to install rope drives for longer distances.

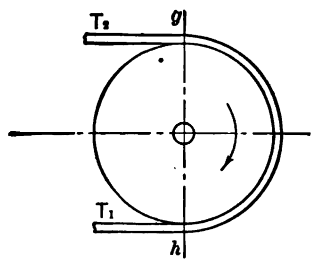


FIG. 9-4.

Horizontal drives are better than vertical. This applies especially to main drives, as the machines in a shop must of necessity have nearly vertical drives. The driving side should be on bottom, as this gives the largest arc of contact. The necessary initial tension may be obtained by any one of three methods: first by making the belt short enough so that it has to be stretched over the pulleys, thus producing the tension. Second, by the use of tightening pulleys or idlers (Fig. 9-7). These should be applied on the slack side of belt. Third, in the case of large belts and long center distances the weight of the belt itself is sufficient to produce the required tension.

When two shafts which are not parallel are to be connected by a belt it may be necessary to use guide pulleys. In any case the following rule must be observed, viz.: the center line of the advancing side of the belt must be in a plane which is perpendicular

to the shaft toward which it is advancing. Fig. 9–5 shows what is termed a quarter twist drive connecting two shafts at right

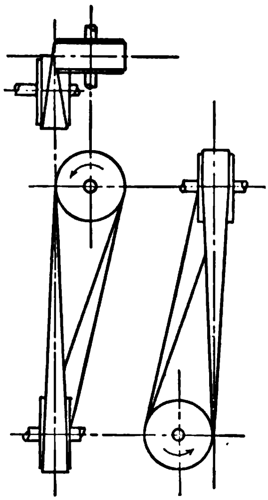


FIG. 9–5.

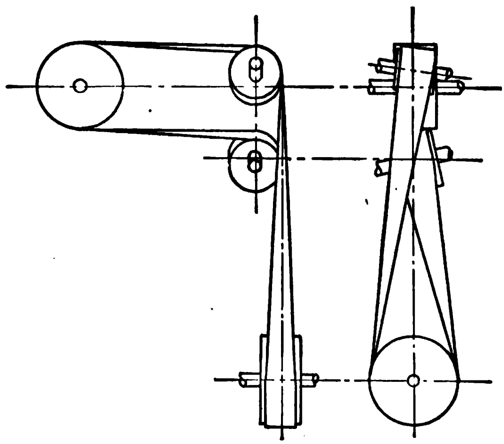


FIG. 9–6.

angles. Here rotation in one direction only is possible. Fig. 9–6 shows a similar drive with the use of two guide pulleys permitting rotation in either direction.

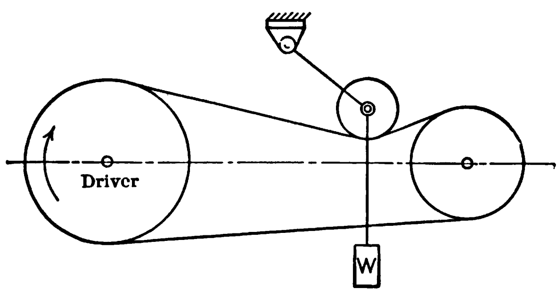


FIG. 9–7.

Creep in Belts. When two pulleys are connected by a belt the theoretical angular velocity ratio of driver to that of driven is $\frac{d+t}{D+t}$, where D is diameter of driver, d diameter of driven, and t is the thickness of belt. Actually the speed of driven pulley will be 2 to 3 per cent less than the theoretical speed. This loss

is due to creep of belt. Creeping results from the elasticity of belt. If there be no slipping of the belt on pulley, its rim will have the same velocity as that of belt where contact begins. If c is the coefficient of extension; that is, the amount a belt 1 ft. long stretches when subjected to a stress of 1 lb. per square inch, then each original foot of belt as it runs off at h (Fig. 9–4) will have a length $1+\frac{T_1}{A}c$ ft. At g this length will be $1+\frac{T_2}{A}c$ ft. Therefore the difference in length of belt running on at g and off at h is $\frac{T_1-T_2}{A}c$. This represents the proportionate loss in speed and reducing to simplest terms

$$\frac{T_1-T_2}{A}=\frac{P}{A}=p,$$

that is, p represents the driving force of belt per square inch of sectional area. Then the proportional loss of speed is

$$L=pcv.$$

This loss of speed must not be confused with slipping of belt due to excessive load.

Pulleys

Material and Construction. The materials used for pulleys are cast iron, steel, wrought iron, wood pulp, and paper, cast iron being by far the commonest material used. They may be solid, that is, in one piece, or split pulleys made in two halves and bolted together. Pulleys running at high speed should be carefully balanced to prevent excessive vibration. The rims may be flat (Fig. 9–8), crowned (Fig. 9–9) or flanged (Fig. 9–10). The flat rim is used where a belt has to be shifted. The purpose of crowning is to make the belt run centrally on pulley. Flanged pulleys are used where the belt may fall off due to excessive slipping or on vertical shafts.

Proportions of Rims, Arms and Hubs. The width, w, of pulley rim (Fig. 9–8) should be greater than that of belt and may be made

$$w=1.1b+\tfrac{1}{2}\text{ in},$$

The amount of crowning is

$$c=\frac{\sqrt[3]{w}}{10}.$$

The thickness of rim at edge may be

$$t=\frac{\sqrt{D}}{20}+\frac{1}{16}\text{ in.},$$

where D is diameter of pulley in inches. The diameter of hub is given by

$$d_1=1.5d+1\text{ in.},$$

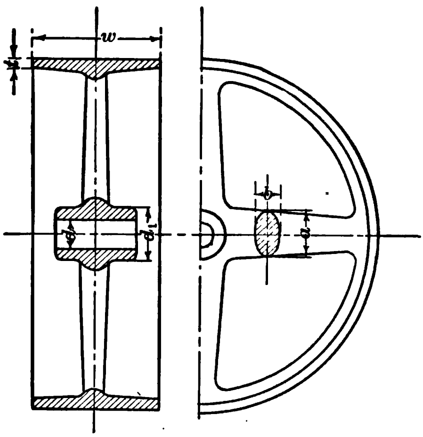


FIG. 9-8.

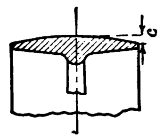


FIG. 9-9.

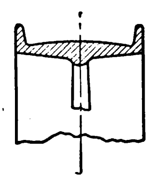


FIG. 9-10.

where d is the diameter of shaft.

The arms may be calculated on the assumption that they are cantilevers subjected to a load equal to the effective belt pull.

Let P = pull of belt at rim,
R = pulley radius in inches,
n = number of arms.

Then we may assume that actually one-half of the arms carry the load P and therefore the bending moment on each arm is

$$M=\frac{PR}{\frac{1}{2}n}=\frac{2PR}{n},$$

$$M=zs;$$

and

$$\therefore\quad z=\frac{2PR}{ns}. \qquad (11)$$

With the usual elliptic section of arm (Fig. 9–8) in which major axis, a, is twice the minor axis, b, the section modulus is

$$z=\frac{\pi}{32}a^2b=\frac{\pi}{64}a^3;$$

and substituting this value in equation (11) we obtain

$$\therefore \quad a=\sqrt[3]{\frac{128PR}{\pi ns}}$$

$$=3.4\sqrt[3]{\frac{PR}{ns}}. \qquad \qquad (12)$$

3.44

This gives the dimensions at center of pulley, the arm being tapered from $\frac{1}{4}$ to $\frac{3}{8}$ in. per foot. However, the dimensions at rim should not be less than two-thirds of those at center. Very broad pulleys are provided with two sets of arms.

Example. Determine dimensions of arms of a cast-iron pulley 48 in. diameter, transmitting 30 H.P. at 150 R.P.M. The pulley has six arms.

$$PR=\frac{63000\times 30}{150}=12600.$$

The safe stress may be taken at from 1800 to 3000, using 2000 we obtain from equation (11)

$$a=3.4\sqrt[3]{\frac{12600}{2000\times 6}}=3\tfrac{1}{2}\text{ in. (nearly).}$$

Tight and loose pulleys are shown in Figs. 9–11. These are used where a machine is to be started and stopped frequently, although in a great many cases a friction clutch is to be preferred for this purpose. The hub of the loose pulley is made quite long to reduce wear. The pulley may run directly on shaft or a cast-iron, steel, brass, bronze or babbitted bushing may be used, as in Fig. 9–12. Here the sleeve is held on shaft by a set screw, it is hollowed out to act as an oil reservoir. Sometimes the diameter of the loose pulley is somewhat less than that of the tight pulley, the purpose being to relieve the tension in belt.

Split pulleys are easy to put in position on shaft. Fig. 9–13 shows a cast-iron split pulley. The weight of steel pulleys is only about one-third that of cast iron. This may be of considerable

advantage for line shafts with many pulleys, since there is a corresponding reduction of bearing friction and wear.

Cone Pulleys are used when a number of speeds are required, as in nearly all machine tools. It is advantageous to have the speeds in geometrical progression.

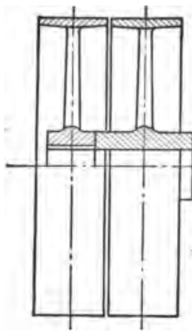


Fig. 9–11.

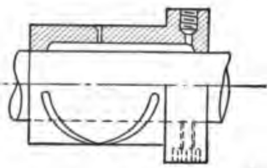


Fig. 9–12.

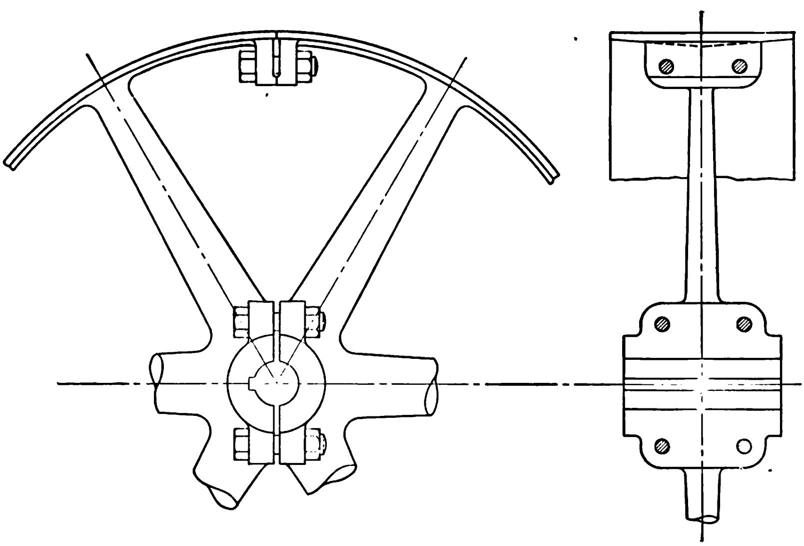


Fig. 9–13.

Let $n_1, n_2, n_3, \dots n_x =$ speeds of driven shaft,

$r =$ ratio of geometric progression,

$x =$ number of steps on cone pulley,

then $n_2 = rn_1,\ n_3 = rn_2 = r^2n$, etc.;

and

$$r = \left(\frac{n_x}{n_1}\right)^{\frac{1}{x-1}}.$$

Since the belt is shifted from one step of the cone to the other it is of course necessary that their diameters be such as to require the same length of belt on each step. Fig. 9–14 gives Burmester's

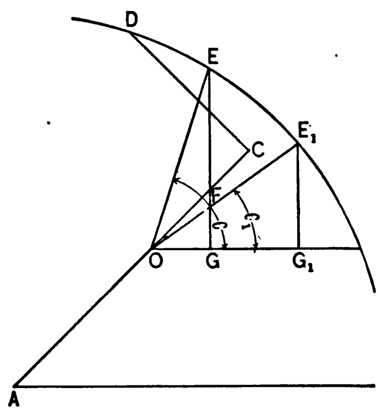


FIG. 9–14.

graphical solution of this problem. The distance between centers of shafts and the radii of the first step of each of the cone pulleys must be known.

Draw the line AC at an angle of 45° to the horizontal line AB and length equal to d, the distance between shafts. Draw CD equal to $\frac{1}{2}d$ and perpendicular to AC. With A as a center and radius AD strike an arc. Locate a point E on this arc so that the vertical distance between E and a point F on AC is equal to $R-r$, the difference of the radii of the first steps of pulleys. Extend this line to G so that FG is equal to r. Through G draw the horizontal line GO, then $EG=R$ and $OG=FG=r$. Draw OE and let angle $EOG=\theta$, then $\tan\,\theta=\frac{EG}{OG}=\frac{R}{r}=$ the velocity ratio m. The radii R_1, and r_1, for any other velocity ratio m_1, may be found by drawing OE_1, so that $\tan\,\theta_1=m_1$, then $EG_1=R_1$ and $OG_1=r_1$.

PROBLEMS

1. A pulley 54 ins. diameter transmits 60 H.P. at 175 R.P.M. The arc of contact is 160°. Find width of $\frac{5}{16}$-in. belt which would be required if coefficient of friction is .35 and a safe stress of 300 lbs. per square inch be permitted.

2. An engine develops 50 H.P. at 200 R.P.M. The belt wheel is 60 ins. diameter and drives through an intermediate jack shaft, a dynamo running at 1200 R.P.M. and having an 18-in. pulley on armature shaft. Find width of both belts assuming $\frac{T_1}{T_2}=2.5$ and allowing a tension of 90 lbs. per inch of width. Sketch the arrangement of pulleys giving their diameters.

3. What horse-power can be transmitted by a belt 4 ins. in width and $\frac{1}{4}$ in. thickness when running over a 26-in. pulley making 250 R.P.M., the coefficient of friction is .38 and the maximum stress not to exceed 300 lbs. per square inch. Arc of contact is 160°.

4. Design and make sketch of a pulley 40 ins. in diameter to transmit 30 H.P. at 125 R.P.M. The pulley bore is 3 ins.

5. An engine band wheel is 84 ins. in diameter. It transmits 100 H.P. at 125 R.P.M. It has 8 arms of the usual elliptic section. Determine dimensions of arms allowing a maximum bending stress of 3000 lbs. per square inch.

CHAPTER X

FRICTION WHEELS

Friction Wheels are applicable for light and medium powers to machinery which is to be frequently stopped and started, also where a wide range of speeds is desired as in the feed mechanism

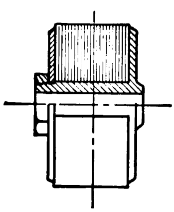


Fig. 10-1.

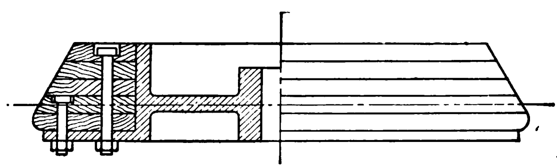


Fig. 10-2.

of various machine tools. For power transmission a fairly high rotative speed is essential. The friction surfaces may be metal, wood, paper, fiber, leather, or rubber. The driver should be of the softer material, the driven being a metal wheel, usually cast iron. Spur, bevel and disc friction wheels are commonly employed as shown in Figs. 10-1 to 10-3.

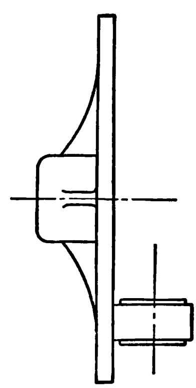


Fig. 10-3.

As the driving capacity of these wheels depends on the friction of the contact surfaces, it is essential to keep these clean and free from grease or oil. They should be rigidly supported by adjacent bearings to maintain an even contact pressure across the entire face of the wheels.

Transmission of Power. The coefficient of friction and the contact pressure determine the amount of power that can be transmitted. The conditions vary so widely that laboratory experiments on the coefficient of friction do not always give a safe value for

their design. In the table below μ is the coefficient of friction and p is the safe contact pressure per inch of width of face.

Material.	μ	p
Cast iron on cast iron	.10–.15	300–500
Wood on cast iron	.20–.50	150–200
Paper on cast iron	.15–.22	150–250
Leather on cast iron	.20–.30	200–300
Rubber on cast iron	.20	
Fiber on cast iron	.20–.30	150–250

Let w = width of cylindrical friction wheel in inches,
μ = coefficient of friction,
D = diameter of wheel in feet,
N = revolutions per minute,
p = safe contact pressure per inch of width,
P = tangential force at rim of wheel,
V = velocity of contact surfaces in feet per minute;

then

$$P=\mu wp$$

and the horse-power transmitted is

$$H=\frac{PV}{33000}=\frac{\mu wp\pi DN}{33000}$$

$$=.000095\ \mu wpND. \qquad (1)$$

It is more usual that the width of a wheel to transmit a given horse-power is required, solving equation (1) for this, we obtain

$$w=\frac{H}{.000095\mu pND}$$

$$=\frac{10500H}{\mu pND}. \qquad (2)$$

Bevel Frictions are suitable only for very light power transmission, as it is difficult to maintain an even contact pressure. The forces Q_1 and Q_2 (Fig. 10–4) which must be exerted along the shafts to engage the wheels and to obtain a driving force P are

$$Q_1=P\frac{\sin\alpha+\mu\cos\alpha}{\mu} \qquad (3)$$

$$Q_2=P\frac{\sin\beta+\mu\cos\beta}{\mu}; \qquad (4)$$

and the horse-power transmitted is

$$H = .000095PND; \quad \ldots \ldots \ldots \quad (5)$$

where D is the mean diameter of either bevel wheel and N the revolutions per minute of the same wheel.

Grooved Frictions are capable of transmitting more power than cylindrical and are therefore used where considerable power

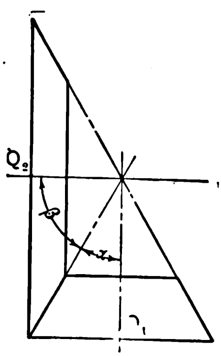


FIG. 10–4.

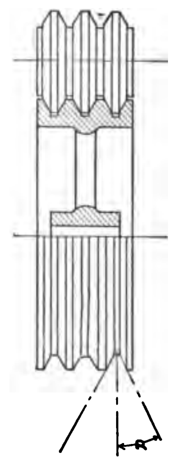


FIG. 10–5.

is required, as in hoisting machines. If α is one-half of the groove angle (Fig. 10–5) then using the same notation as before

$$Q = P\frac{\sin\alpha + \mu\cos\alpha}{\mu}. \quad \ldots \ldots \ldots \quad (6)$$

The usual values of α are 12° to 20°. For $\alpha = 15°$ and two cast-iron wheels with $\mu = .12$.

$$Q = 3.2P.$$

The efficiency of this class of friction wheels may be taken at from 85 to 90 per cent. As there can be rolling motion at one point only on each line of contact there is considerable wear. To reduce this as far as possible the depth of groove should be small, from $\frac{3}{8}$ to $\frac{3}{4}$ in.

Methods of Engaging Friction Wheels. Fig. 10–6 shows the method of engaging two spur frictions. The bearing sleeve A is bored eccentrically and may be rotated through a small angle

by means of the attached lever. In this way the center C of the shaft is shifted and the two wheels may be pressed together. For bevel wheels the bearing sleeve, A, has a long pitch thread cut on its outside as shown in Fig. 10–7. This sleeve may be turned

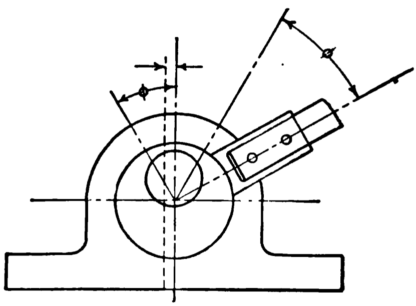


FIG. 10–6.

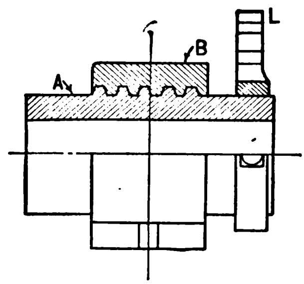


FIG. 10–7.

by means of the lever L, thus moving it in the direction of the shaft axis. The axial motion of sleeve is transmitted directly to bevel wheel or to shaft by means of set collars.

PROBLEMS

1. Two shafts 24 ins. between centers are connected by a pair of friction wheels. The driver makes 240 R.P.M., the follower making 120 R.P.M. Find width of friction if 15 H.P. is to be transmitted. Let the pressure be 150 lbs. per inch of width and the coefficient of friction $= .2$.

2. A small printing press is driven by a 3-H.P. motor running at 1200 R.P.M. On the motor shaft is a rubber friction wheel 5 ins. diameter. Find width of friction wheel if the pressure is 100 lbs. per inch.

3. A pair of grooved frictions is used to connect two shafts 33 ins. between centers. The driver makes 240 R.P.M. and the follower 200 R.P.M. Find pressure on bearings if 12 H.P. is to be transmitted and $\mu = .12$, angle $\alpha = 15°$.

4. In problem 3 if each wheel has six grooves $\frac{1}{2}$ in. deep, what is the contact pressure per inch of contact line?

5. Find horse-power that can be transmitted by a grooved friction wheel 18 ins. diameter when running at 250 R.P.M. The wheels are pressed together with a force of 1000 lbs. and the coefficient of friction is .2. The angle of grove (α) is 15°.

6. A certain friction hoisting machine has two driving wheels $7\frac{1}{2}$ ins. diameter. The two followers, which are 44 ins. diameter, are keyed to the drum shaft. The faces are 6 ins. Determine the load which can be hoisted if the drum diameter is 23 ins. Assume pressure per inch of face is 250 lbs. and coefficient of friction is .2. What horse-power is required if the drivers make 225 R.P.M.?

CHAPTER XI

TOOTHED GEARS

Spur Gears. Toothed gearing is used to transmit motion between shafts which are comparatively close together, or where a definite velocity ratio between the shafts must be accurately maintained, as in the screw-cutting mechanism of a lathe. The imaginary surfaces upon which the teeth are formed are called pitch surfaces. The gears connecting parallel shafts have cylindrical pitch surfaces and are called spur gears.

The object of the teeth is to obtain a positive transmission of motion. The tooth profiles must be such that the motion transmitted by them will give the same velocity ratio as would be obtained by rolling the pitch surfaces upon each other. It is assumed that the reader is familiar with the methods of drawing gear teeth of either the involute or the cycloidal systems. Gear teeth may be cast or cut by properly formed cutters. Since cast gears are more or less inaccurate they are not suitable for high speeds on account of excessive noise and vibration.

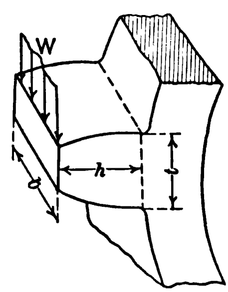


FIG. 11-1.

Strength of Gear Teeth. In the following method of calculation a number of assumptions are made which are more or less approximately correct. In Fig. 11-1 let

W = load acting on tooth in pounds,
p = circular pitch of gear,
b = breadth (face) of gear in inches,
h = height of tooth $= .7p$,
t = thickness of tooth $= .5p$.

The tooth acts as a cantilever carrying the load W at its end, then the bending moment is

$$M = Wh = .7pW.$$

The section at which this bending moment acts is a rectangle of height t and breadth b.

$$M = sz,$$

$$.7pW = s\tfrac{1}{6}bt^2 = s\tfrac{1}{6}b\frac{p^2}{4};$$

and

$$\therefore \quad p = \frac{16.8W}{sb}. \qquad (1)$$

It is usual to assume the breadth, b, of the teeth in terms of the pitch; that is, $b = np$ where n is a constant whose value depends on the speed of the gears. Then

$$p = \frac{16.8W}{snp};$$

$$\therefore \quad p = 4.1\sqrt{\frac{W}{ns}}. \qquad (2)$$

For very low speeds as in hand-operated machines, $n = 1\tfrac{1}{2}$ to $2\tfrac{1}{2}$, for medium and high speeds $n = 3$ to 6. In very high-speed machinery such as steam turbines where herringbone gears are used the value of n may be from 20 to 30, or more.

It frequently happens that the load W is not known, but the twisting moment T and the number of teeth, N, are known. Then if R is the pitch radius of the gear in inches

$$T = WR;$$

but

$$R = \frac{Np}{2\pi};$$

$$\therefore \quad W = \frac{T}{R} = \frac{2\pi T}{Np}.$$

If this value of W be substituted in equation (2) we obtain

$$p = 4.1\sqrt{\frac{2\pi T}{nNps}}$$

$$= 4.72\sqrt[3]{\frac{T}{nNs}}. \qquad (3)$$

If the horse-power, H, to be transmitted and the speed, ω, in revolutions per minute, are known, then since

$$H=\frac{2\pi R\omega W}{33000\times 12};$$

$$\therefore \quad W=\frac{H\times 33000\times 12}{2\pi R\omega};$$

substituting for R its value $\frac{Np}{2\pi}$,

$$W=\frac{H\times 33000\times 12}{Np\omega}.$$

This value of W may now be substituted in equation (2) and we obtain

$$p=4.1\sqrt{\frac{33000\times 12\times H}{pnsN\omega}}$$

$$=188\sqrt[3]{\frac{H}{nsN\omega}}. \qquad (4)$$

In deducing the above equations it has been assumed that the entire force comes on one tooth and that it acts at right angles to the radial plane through center of tooth. A third assumption, that the thickness of tooth at the root is equal to one-half of the

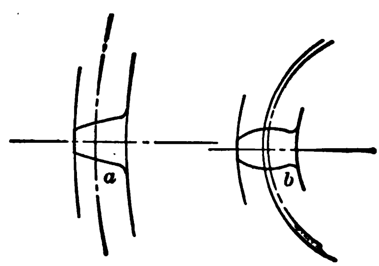


FIG. 11-2.

pitch is true only for one wheel in an interchangeable set. The tooth thickness being less in pinions having few teeth and greater in gears having many teeth. This is shown in Figs. 11-2, where a is the tooth of a 96-tooth gear, and b that of a 10-tooth pinion.

EXAMPLE. Design the gears for a hand-operated hoist to lift a load of 1500 lbs., assuming drum to be 8 ins. in diameter (Fig. 11–3).

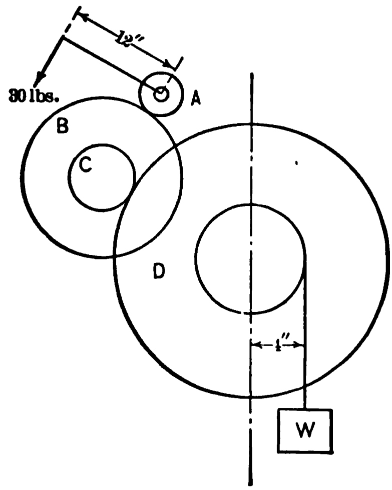


FIG. 11–3.

If the crank be 12 ins. long and the force exerted on crank handle is 30 lbs. the gear train ratio, neglecting friction, is

$$g=\frac{4\times 1500}{30\times 12}=\frac{16.6}{1}.$$

Using four gears this ratio may be divided into two factors or

$$g=\frac{16.6}{1}=\frac{4}{1}\times\frac{4.15}{1};$$

and the number of teeth may now be assumed according to this ratio

$$\frac{D}{C}\times\frac{B}{A}=\frac{83}{20}\times\frac{48}{12}.$$

Equation (3) will be applicable to this case. Assume $n=2$ and $s=6000$, then since for gear D the twisting moment is

$$T=4\times 1500=6000 \text{ in.-lbs.},$$

$$p=4.72\sqrt[3]{\frac{6000}{2\times 83\times 6000}}$$

$$=.86=\text{say } \tfrac{7}{8} \text{ in.}$$

For gear B the twisting moment is

$$T' = \frac{6000}{4.15} = 1445 \text{ in.-lbs.};$$

and

$$p = 4.72\sqrt[3]{\frac{1445}{2\times48\times6000}}$$

$$= .64 \text{ in.} = \text{say } \tfrac{5}{8} \text{ in.}$$

The Lewis Equation. This equation for the strength of gear teeth deduced by W. Lewis takes into consideration both the form of the tooth and the obliquity of the line of pressure. It is widely used by American designers. Using the same notation as previously this equation, for the 15° involute, and for the cycloidal system in which the 12-tooth pinion has radial flanks is

$$W = spb\left(.124 - \frac{.684}{N}\right); \quad \ldots \ldots \quad (5)$$

and substituting for b its value np we obtain

$$p = \sqrt{\frac{W}{ns\left(.124 - \frac{.684}{N}\right)}}. \quad \ldots \ldots \quad (6)$$

If the twisting moment T to be transmitted is known we have

$$W = \frac{2\pi T}{Np};$$

and substituting this value of W in (6),

$$p = 1.84\sqrt[3]{\frac{T}{nsN\left(.124 - \frac{.684}{N}\right)}}. \quad \ldots \quad (7)$$

The values of the safe stress s for any material depends to a great extent on the speed of the gears and on the steadiness of the load. The following table gives safe values for gears that are well supported in rigid bearings. For excessive shock, as in rock-crushing machinery or rolling-mill work, these values may be reduced.

Velocity of Pitch Line, Feet per Minute.	100 or Less.	200	300	600	1000	1500	2000
s for cast iron.	6,000	4,500	3,600	3000	2500	2000	1800
s for steel.	15,000	12,000	10,000	9000	8000	6000	5000

For the cycloidal system in which the 15-tooth pinion has radial flanks the Lewis equation is

$$W = spb\left(.106 - \frac{.678}{N}\right). \qquad (8)$$

Bevel gears are used to connect two shafts which lie in the same plane but are not parallel to each other. Such gears have conical pitch surfaces. Fig. 11–4 shows a pair of bevel gears to

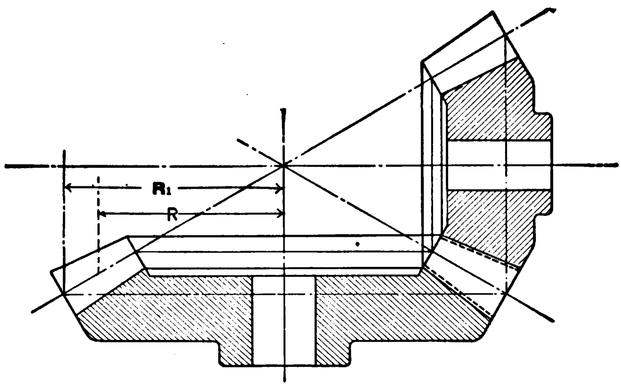


FIG. 11–4.

connect two shafts at right angles. The pitch of these gears may be calculated in the same way as that of spur gears. The value thus found is to be taken as the mean pitch half way between the large and the small end of the teeth at radius R. The actual pitch at large end is $\frac{R_1}{R}$ times the calculated pitch.

Spiral gears are used to connect two shafts that are neither parallel nor in the same plane. Fig. 11–5 shows a pair of such gears. The tooth elements here are helices and therefore the gears are in point contact, theoretically. For this reason the

strength of the tooth is rarely a factor in their design as the contact pressure limits the safe load.

Worm Gears. The worm and worm wheel may be used where a large speed reduction is required; the shafts usually are at right angles. The pitch of worm wheel is calculated as that of a spur gear. However since in worm gears there are always a number of pairs of teeth in contact, it will be safe to assume that the load on teeth to be used in the calculation for pitch is $\frac{1}{2}$ of the total tangential force acting at pitch line of worm wheel.

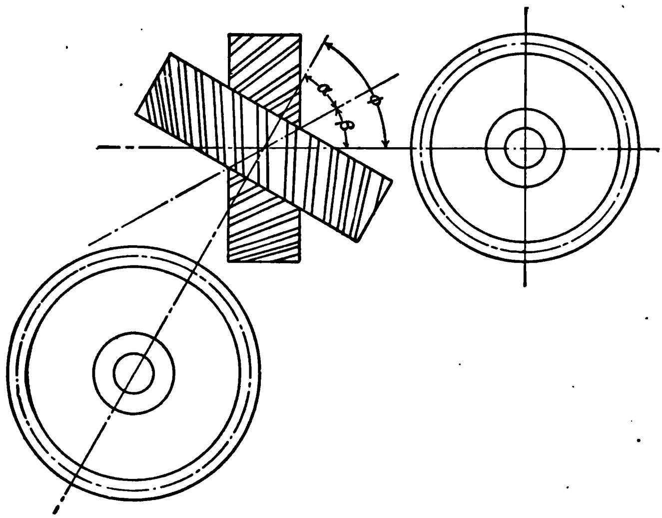


FIG. 11-5.

Fig. 11-6 shows a section through a pair of these gears. Since the teeth slide over each other with a velocity equal to that of the pitch line of the worm, wear is an important consideration in their design. To limit the wear it is therefore necessary to reduce the contact pressure with increasing speed.

The efficiency of worm gearing depends on the angle of the worm thread, that is on the angle of the pitch helix of worm.

Let E = efficiency,

α = angle of worm thread,

μ = coefficient of friction,

d = pitch diameter of worm,

$\beta = \tan^{-1} \mu$;

then if the friction of bearings be neglected

$$E=\frac{\tan \alpha}{\tan (\alpha+\beta)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From this equation it is evident that $E=0$ when $\alpha=0$ and also when $\alpha=90°-\beta$. The efficiency being a maximum when $\alpha=\frac{90°-\beta}{2}$. For cast-iron worm wheel and a steel worm running

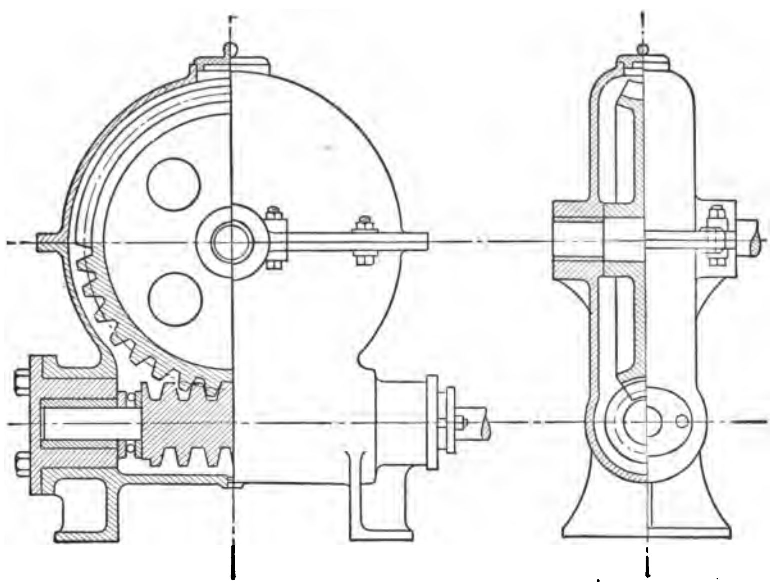


FIG. 11-6.

in oil and good workmanship, μ may be taken at .05. This gives for maximum efficiency $\alpha=43° \ 30'$ approximately. So large an angle cannot be obtained in practice and fortunately the efficiency is very satisfactory, if α is from 15° to 20°, a value which may be obtained by proper design. Since $\tan \alpha=\frac{p}{\pi d}$ it is evident that d should be made as small as possible in order to obtain the maximum efficiency for a given pitch.

THEORETICAL EFFICIENCY

μ	α					
	5°	10°	15°	20°	25°	30°
.02	81.3	89.5	92.6	94.1	95.0	95.5
.04	68.4	80.9	86.1	88.8	90.4	91.4
.06	59.0	73.8	80.4	84.0	86.1	87.5
.08	51.9	67.8	75.4	79.6	82.2	83.8

Gear Rims and Arms. The thickness, t, of gear rims may be made $.5p+\frac{1}{4}$ in. for gears of small pitch and $.4p$ for gears of large pitch. Fig. 11–7 shows the common sections of gear arms. They may be calculated in the same way as the pulley arms in Chapter IX, except that the assumption is made that only one-third of the arms carry the load. This changes equation (11) of that chapter to

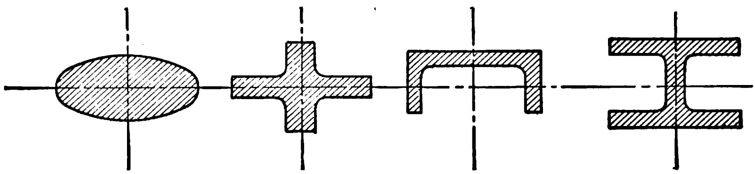


FIG. 11–7.

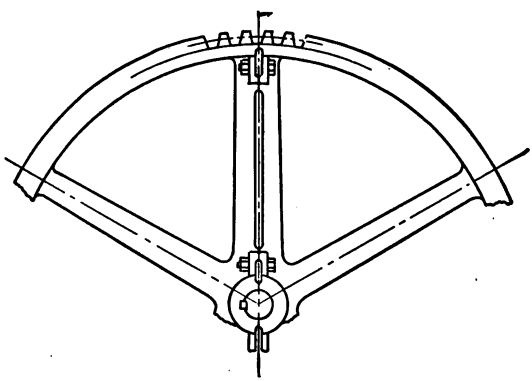


FIG. 11–8.

$$z=\frac{3PR}{ns}, \quad \ldots \ldots \quad (10)$$

from which the proportions of the arms at the hub may be calculated. The diameter of the hub may be $D=1.75d+\frac{1}{2}$ in. where d is the shaft diameter.

Split Gears. Large and heavy gears are frequently cast in halves and these are bolted together or shrink links are used. Fig. 11–8 shows a portion of such a gear which is parted along the center of an arm. The bolts should be calculated so that those at the rim alone will be capable of sustaining the entire force acting at the teeth. In very heavy gears both shrink links and bolts are used.

PROBLEMS

1. A gear having a pitch diameter of 6 ins. is to transmit 5 horse-power at 120 R.P.M. Determine its pitch if a stress of 4000 lbs. per square inch be permitted and face of gear is $3\frac{1}{2}$ times the pitch.

2. Two shafts running at 90 and 126 R.P.M. and transmitting 70 H.P. are connected by spur gears. The distance between centers is to be as near 60 ins. as possible. Find pitch of gears, assuming face to be three times the pitch and allowing a stress of 4500 lbs. per square inch.

3. A gear has 40 teeth of 2 diametral pitch and 4 ins. face. It transmits a twisting moment of 15,000 in.-lbs. Determine stress in teeth.

4. A punch press running at 40 R.P.M. is driven by a 10-H.P. motor running at 1200 R.P.M., a double-gear reduction being used. Design the gear train, state all necessary assumptions. Make a sketch of mechanism.

5. Design the arms for gears in problem 2, assume each gear has six arms and that the stress is 3500 lbs. per square inch.

6. The gate of a sluice valve weighing 10,000 lbs. is raised by means of a rack and pinion. Design a train of gears so that gate may be lifted by two men working on a 14-in. crank handle and exerting a pressure of 40 lbs. each.

7. The drum of a hoist is 16 ins. in diameter. The gear on same shaft with drum is 36 ins. in diameter and meshes with a pinion 6 ins. in diameter. On same shaft with this pinion is a gear 24 ins. in diameter which meshes with another pinion 6 ins. in diameter. The capacity of hoist is 3000 lbs. Determine pitch and number of teeth on each gear. Assume stress at 5000 lbs. and face of each gear $2\frac{1}{2}$ times the pitch.

8. An 18-tooth cast-iron pinion is to transmit 15 H.P. at 200 R.P.M. Determine the pitch required by means of the Lewis equation. Assume $n=3\frac{1}{2}$.

9. In a worm gear of 1-in. pitch, the worm is double threaded and has a pitch diameter of $2\frac{1}{2}$ ins. Determine efficiency of this gear. Assume cast-iron worm wheel and steel worm, $\mu=.05$.

CHAPTER XII

ROPE TRANSMISSION

Textile Ropes

Textile ropes are used very extensively for the transmission of power in cotton, steel and flouring mills, both for main drives and for individual machines. For the transmission of medium and large powers, from 250 H.P. up, and where the distance between shafts is considerable, rope drives possess many advantages. Among these may be mentioned economy in first cost and maintenance, noiseless and steady running, small space required on shaft by rope sheaves and ease with which power may be divided and transmitted to various floors of a building.

Rope Materials. Cotton, hemp and manila hemp are the materials most widely used in the manufacture of transmission rope. Cotton makes the most flexible rope and is therefore better adapted for driving individual machines where centers between shafts are short and lack of space prohibits the use of large rope pulleys. Manila fiber is considerably stronger than cotton and is largely used for heavy main drives. The rope material is laid into strands and these are twisted together to form the rope. Three, four, and sometimes six strands are used for transmission rope. Figs. 12–1 and 12–2 show sections of such ropes. It will be noted that the four-strand rope is laid about a central core and is of a more nearly circular section; it will therefore give somewhat better surface wear than the three-strand rope. The sectional area of this rope is approximately $.64d^2$.

Systems of Rope Driving. There are two systems of rope driving, known as the multiple, or English, and the continuous, or American system. Although engineers differ widely as to the relative merits of these systems, there can be no doubt that each has its field of usefulness. In the multiple system a number of independent ropes are used side by side, sufficient to transmit

the required power. In the continuous system a single endless rope passes from the first groove of the driver to the first groove of the driven back to the second groove of driver, and so on continuously to the last groove of driven from which it is returned

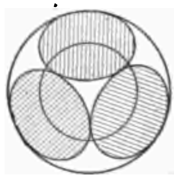


FIG. 12-1.

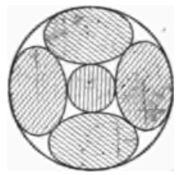


FIG. 12-2.

to first groove of driver by means of one or more guide pulleys One of these guide pulleys is placed in a weighted movable frame called a tension carriage, by means of which sufficient tension is put on the ropes to prevent their slipping in grooves of pulleys when transmitting power.

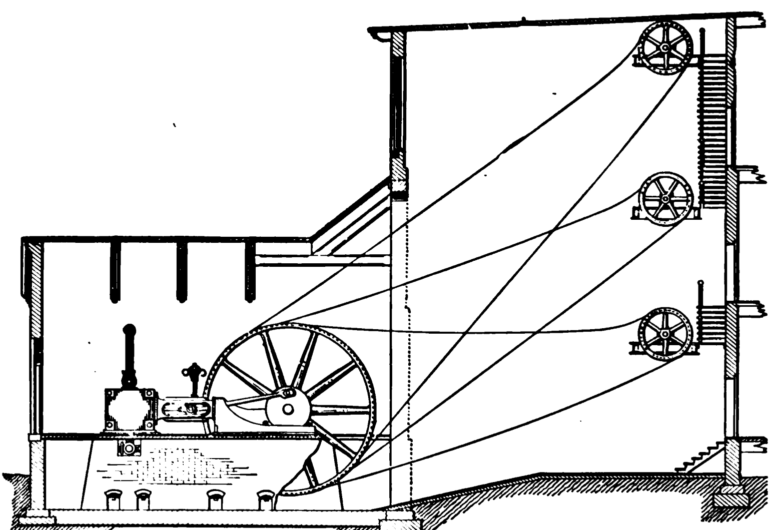


FIG. 12-3.

The multiple system is especially adapted for large main drives if horizontal or nearly so. The difficulty is to get each rope to transmit its proportionate part of the entire load. To do this

the grooves must be of the same diameter, the ropes must be uniformly spliced and be put on with the same tension. With sudden variations in load the ropes tends to jump out of their grooves. To prevent this many engineers use deeper grooves than for the

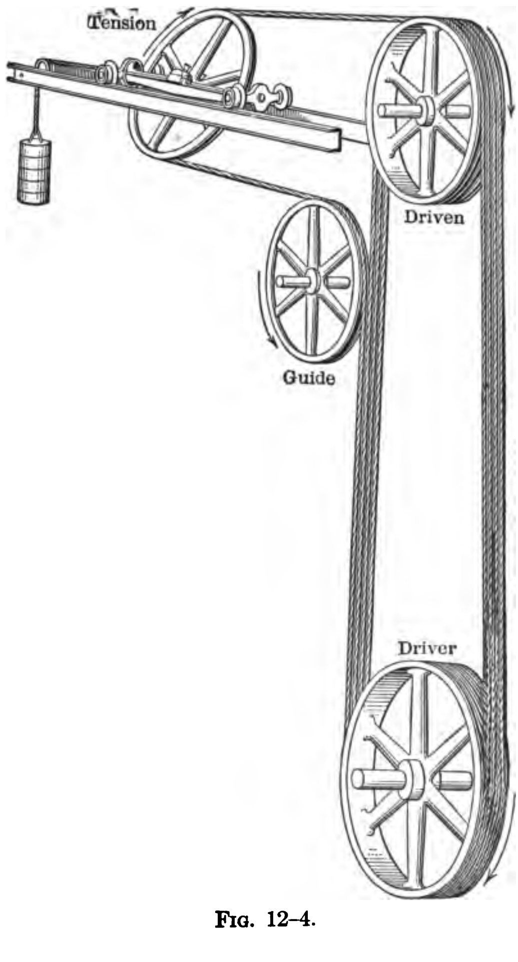


Fig. 12–4.

continuous system. There is greater freedom from breakdowns with this system, as the breaking of one rope does not cause shut down. It is also somewhat cheaper to install. The continuous system is much used for small and medium powers, for driving individual machines, and for exposed drives. This

system is distinguished by its great flexibility, as it easily adapts itself to difficult conditions such as vertical drives and quarter twist drives. The tension carriage, if properly installed, maintains at all times a uniform tension on each turn of the rope. It automatically takes up the stretch in the rope and therefore changes in atmospheric conditions do not affect such a drive. Fig. 12–3 shows the distribution of power in a mill by means of the multiple system, while Fig. 12–4 shows a vertical drive using the continuous system.

Rope Pulleys. In order to secure sufficient adhesion for driving the rope pulley is provided with grooves. Many different forms of grooves have been used in which the angle between the sides varies from 30° to 75°. Fig. 12–5 shows a groove

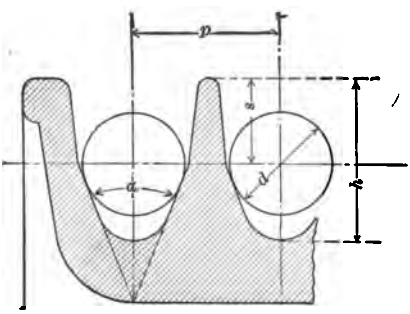


FIG. 12—5.

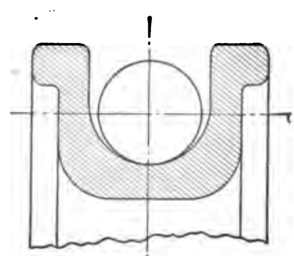


FIG. 12–6.

which gives average American practice. The proportions are $p = 1\frac{1}{4}d + \frac{1}{2}''$, $s = \frac{3}{4}d$, $h = 1\frac{5}{8}d$ and $\alpha = 45°$. This value of the angle α is well-nigh universal for ordinary conditions of driving. As no wedging action is desired in guide pulleys the grooves of these are made so that the rope rests on its bottom as shown in Fig. 12–6.

Since the greatest wear on a rope is internal, due to the rubbing of the fibers over each other as the rope is bent around a pulley, the diameter of these should be as large as practicable. It is customary to make the minimum pulley diameter for cotton ropes thirty times the rope diameter, and for manila forty times the rope diameter. The external wear of rope is due to slipping. To minimize this the grooves must be turned accurately to the same diameter and outline. They should be finished smooth by

polishing. As rope pulleys run at a comparatively high rate of speed they should be carefully balanced.

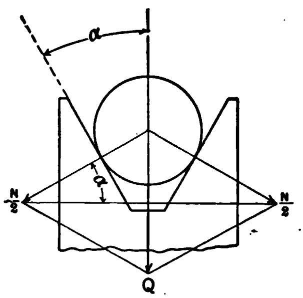


FIG. 12–7.

Power Transmission. The driving capacity of a rope like that of a belt depends upon the friction between rope and pulley and therefore the method of deducing the power transmitted is similar to that of Chapter IX. Since the rope lies in a groove a radial force Q (Fig. 12–7) produces a normal pressure, N, between the rope and the sides of groove, this pressure is given by

$$N=\frac{Q}{\sin\alpha}, \qquad (1)$$

and the friction is

$$F=\mu N=\frac{Q\mu}{\sin\alpha}, \qquad (2)$$

that is, the effect of the groove is to increase the value of the coefficient of friction from μ to $\frac{\mu}{\sin\alpha}$. If the symbol, μ', be substituted for this we have just as in equation (3) of Chapter IX,

$$\frac{T_1}{T_2}=e^{\mu'\theta}. \qquad (3)$$

The value of μ' may conservatively be taken at .4, which gives

$$\frac{T_1}{T_2}=3.5. \qquad (4)$$

Since ropes run at comparatively high speeds the effect of centrifugal force cannot be neglected.

Let W = weight of 1 in. of rope,
V = velocity in feet per second,
$g = 32.2$ = acceleration of gravity,
C = centrifugal force per inch of rope,
C_t = tension in rope due to centrifugal force,
R = radius of pulley in inches;

then

$$C=\frac{Wv^2}{g\frac{R}{12}};$$

and

$$C_t=\frac{12Wv^2}{g}. \qquad (5)$$

The weight of transmission rope may be taken at $.34d^2$ per foot, this gives

$$C_t=.0105d^2v^2. \qquad (6)$$

Using the same notation as for belts, we have

$$T_1-T_2=P;$$

but the maximum tension which occurs is

$$T=T_1+C_t;$$

and therefore

$$T_1=T-C_t,$$

$$T-(T_2+C_t)=P. \qquad (7)$$

The average breaking strength of manila rope is $7000d^2$ and of cotton rope $5000d^2$. In order to give long life to transmission rope its factor of safety is very high, thus the value of the maximum allowable stress, T, for manila rope is $200d^2$ and for cotton $180d^2$.

Let H = horse-power transmitted by one rope,
V_m = velocity of rope in feet per minute;

then

$$H=\frac{PV_m}{33000}$$

$$=\frac{(T_1-T_2)V_m}{33000}. \qquad (8)$$

Substituting for T_2 its value $\frac{T_1}{3.5}$ from equation (4),

$$H=\frac{5}{7}\frac{T_1V_m}{33000};$$

but since

$$T_1=T-C_t,$$

$$H=\frac{5}{7}\left\{\frac{(T-C_t)V_m}{33000}\right\}. \qquad (9)$$

For manila rope if the values given above for T and C_t be substituted this equation becomes

$$H=\frac{\frac{5}{7}(200d^2-.0105d^2v^2)V_m}{33000}$$

$$=\frac{(200-.0105v^2)d^2V_m}{46200} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

Example. How many 2-in. ropes are required to transmit 500 H.P. at a rope speed of 3600 ft. per minute.

From equation (10) the horse-power transmitted by one rope is

$$H=\frac{(200-.0105\times3600)4\times3600}{46200}$$

$$=50 \text{ nearly};$$

and therefore the number of ropes necessary is $\frac{500}{50}=10$.

If equation (10) be plotted it will be noticed that the maximum horse-power is reached at a speed of about 5000 ft. per minute and beyond this speed the power decreases, due to the effect of centrifugal force. In Fig. 12–8 the equation has been plotted for a 2-in. rope.

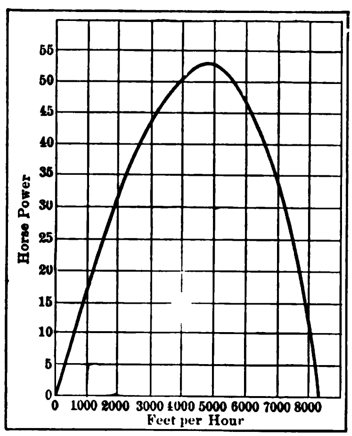


Fig. 12–8.

Wire-rope Transmission. The use of wire rope for the transmission of power alone has a limited field to-day, electric transmission having taken its place; but for hoisting and conveying of material it finds very extensive application. The materials used for the wires of these ropes are Swedish iron or steel. The steel is a high-grade cast steel, the tensile strength of which in the form of wire is from 150,000 to 250,000 lbs. per square inch. The rope commonly used for power transmission consists of six strands laid about a hemp or wire core. Each strand contains seven or nineteen wires, as shown in Figs. 12–9 and 12–10. The

diameter of the wires used depends of course on the number of wires in each strand and may be taken approximately at one-

FIG. 12-9.

ninth rope diameter for the seven-wire strands and one-fifteenth rope diameter for the nineteen-wire strand. As the flexibility

FIG. 12-10.

of a rope depends on the size of the wires used, it is evident that the nineteen-wire strands give a more pliable rope and where great flexibility is desired, a thirty-seven-wire strand is sometimes used.

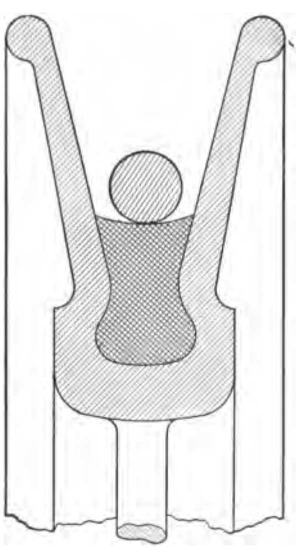


FIG. 12-11.

Wire-rope Pulleys. These are usually made of cast iron and as light as consistent with considerations of strength. Large pulleys may have wrought-iron arms set into cast-iron hubs and rim. They are grooved as in textile-rope transmissions; but unlike such ropes the wire rope is not wedged in between the sides of the groove, as that would cause rapid destruction. The rope rests on bottom of groove and a filling of some soft material is used, such as rubber, wood or leather (Fig. 12-11). The diameter of these pulleys is comparatively large in order to reduce the stress due to bending around the pulley. The ratio of pulley diameter, D, to rope diameter, d, may be taken as follows: for

steel rope 7 wires to the strand $\frac{D}{d}=80$, for 19 wire strand $\frac{D}{d}$ $=50$; for iron wire these values should be doubled.

Transmission of Power. The minimum distance between shafts is about 60 ft. The maximum distance depends on the topography of the ground, for horizontal transmission it is about 600 ft. Across a river or valley this distance may be very much greater. For very long distances intermediate carrying sheaves are used, or a series of spans are employed, the intermediate pulleys having two grooves. If the transmission is not in a straight line guide pulleys or bevel gears may be used at the angles.

In wire transmission rope the stresses are, first, that due to the weight of rope (s_w), second, that due to bending of rope around pulleys (s_b) and third, that the due to centrifugal force (s_c). The sum of these stresses must not exceed the safe stress (s) of the rope. It is only the first of these which produces adhesion of the rope to the pulley and therefore is a measure of the trans mission capacity of the rope. We then have

$$s_w+s_b+s_c=s$$

and, using the same notation as for textile ropes

$$\frac{T_1}{T_2}=e^{\mu\theta},$$

$$T_1-T_2=P.$$

For a rope making a half lap around a rubber or leather-filled pulley, slightly greasy, the coefficient of friction, μ, may be taken at .23, substituting this value we obtain

$$\frac{T_1}{T_2}=e^{.23\times\pi}=2.$$

The adhesive tension, T_1, is due to the weight of the rope and depends on the curve in which it hangs.

$$T_1=\frac{\pi}{4}\,\delta^2 n s_w,$$

where δ = diameter of the individual wires and n is the number of wires in the rope.

The stress due to bending of rope is, according to Bach

$$s_b = \tfrac{3}{8} E \frac{\delta}{D};$$

where E is the modulus of elasticity. The centrifugal stress may be obtained similarly to that of belts. The weight of ordinary wire rope with hemp core is about $1.6d^2$ lbs. per foot; therefore

$$s_c = .13 V_s^2.$$

V_s is the speed of rope in feet per second. This stress is quite small in comparison to s_w and s_b, it may in most cases therefore be neglected without serious error. The safe stress, s, should not exceed one-seventh of the breaking strength. The following example will show the applications of these equations.

Example. What horse-power may be transmitted by a $\tfrac{1}{2}$-in. rope (steel) six 19-wire strands. The sheaves are 5 ft. diameter and the rope runs at 4200 ft. per minute.

$$s_b = \frac{3}{8} E \frac{\delta}{D} = \frac{3}{8} \times 29000000 \times \frac{\frac{1}{15} \cdot \frac{1}{2}}{60}$$

$$= 6050 \text{ lbs. per square inch,}$$

$$s_c = .13 V_s^2 = .13 \times 4900 = 640 \text{ lbs. per square inch.}$$

The safe stress s may be taken at 25000 lbs. per square inch, then

$$s_w = s - (s_b + s_c) = 17{,}860 \text{ lbs. per square inch.}$$

The rope contains 114 wires of about $\tfrac{1}{30}$ in. diameter, then

$$T_1 = \frac{\pi}{4} (\tfrac{1}{30})^2 \times 114 \times 17860 = 1780 \text{ lbs.}$$

$$\therefore \quad P = T_1 - T_2 = \tfrac{1}{2} T_1 = 890 \text{ lbs.};$$

and

$$\text{H.P.} = \frac{PV}{33000} = \frac{890 \times 4200}{33000} = 102.5.$$

Deflection of Ropes. The towers which support the sheaves in a long-distance transmission must be sufficiently high so that rope

may clear the ground. The curve in which rope hangs is approximately a catenary, but it will greatly simplify calculation and introduce no appreciable error if a parabola be substituted. Then the sag (Fig. 12–12) is

$$h=\frac{L^2W}{8T};$$

where

h = sag in feet,
L = span in feet,
W = weight in pounds of 1 ft. of rope,
T = tension in rope.

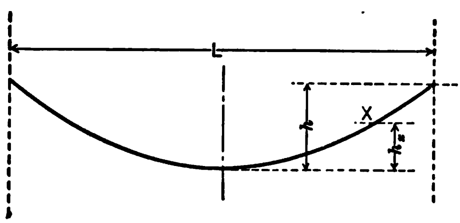


FIG. 12–12.

In the above example the sag in driving side of rope, if a span of 300 ft. be assumed, is

$$h=\frac{\overline{300}^2\times 1.6}{4\times 8\times 1780}=2.5 \text{ ft.};$$

for slack side of rope

$$h=\frac{\overline{300}^2\times 1.6}{4\times 8\times 890}=5 \text{ ft.}$$

If the points of support are not in the same horizontal plane then the following equations will give the amount and position of maximum sag as in Fig. 12–13.

$$L_1=\frac{L}{2}-\frac{TD}{LW},$$

$$L_2=\frac{L}{2}+\frac{TD}{LW},$$

$$h_1=\frac{WL_1^2}{2T},$$

$$h_2=\frac{WL_2^2}{2T}.$$

The tension T is that at the lowest point of rope. At any other point x the tension is T plus the weight of a piece of rope of length h_x or

$$T_x = T + h_x w.$$

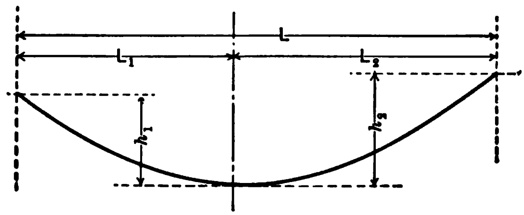


FIG. 12–13.

The following table gives the horse-power which steel rope may safely transmit as recommended by the Trenton Iron Co. This is for wood-filled pulleys having approximately the diameters previously given.

TABLE OF HORSE-POWER

Diam. of Rope.	Velocity of Rope in Feet per Second.									
	10	20	30	40	50	60	70	80	90	100
$\frac{1}{4}$	4	8	13	17	21	25	28	32	37	40
$\frac{5}{16}$	7	13	20	26	33	40	44	51	57	62
$\frac{3}{8}$	10	19	28	38	47	56	64	73	80	89
$\frac{7}{16}$	13	26	38	51	63	75	88	99	109	121
$\frac{1}{2}$	17	34	51	67	83	99	115	130	144	159
$\frac{9}{16}$	22	43	65	86	106	128	147	167	184	203
$\frac{5}{8}$	27	53	79	104	130	155	179	203	225	247
$\frac{11}{16}$	32	63	95	126	157	186	217	245		
$\frac{3}{4}$	38	76	103	150	186	223				
$\frac{7}{8}$	52	104	156	206						
1	68	135	202							

PROBLEMS

1. An engine developing 2000 H.P. at 110 R.P.M. has a rope wheel 12 ft. in diameter. Determine how many 2-in. manila ropes are required for this transmission.

2. In a certain factory 3200 H.P. was to be distributed to four floors as follows: 1st floor 1600 H.P., 2d floor 700 H.P., 3d floor 500 H.P., and 4th floor 400 H.P.; $1\frac{1}{2}$-in. ropes were used. The rope wheel of engine is 18-ft. in diameter and runs at 90 R.P.M. Find the number required for each floor.

3. An engine developing 600 H.P. at 140 R.P.M. has a rope wheel 10 ft. in diameter. It is connected to a jack shaft, running at 220 R.P.M., by means of 2-in. ropes. From the jack shaft the power is transmitted by $1\frac{1}{2}$-in. ropes as follows: 280 H.P. to first floor, 140 H.P. to second floor, and 180 H.P. to third floor; these ropes having a velocity of 3800 ft. per min. Find number of ropes required for each drive.

4. A $\frac{3}{4}$-in. diameter steel wire rope connects two sheaves 60 ins. in diameter and 200 ft. between centers, the velocity of rope being 3600 ft. per minute. The rope has six 7-wire strands, the wire diameter being .08 in. Find the horse-power which can be transmitted.

CHAPTER XIII

CHAIN GEARING

THE transmission of power by means of a chain finds its most extensive application in motor cars and in the individual driving of machine tools by means of electric motors. It is largely due to the bicycle that transmission chains have reached

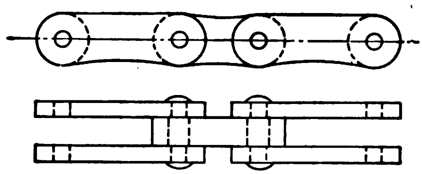


FIG. 13-1.

their present state of perfection. The types of chain used for power transmission are the block chain (Fig. 13-1), the roller chain (Fig. 13-2), and the so-called high-speed or inverted tooth chains, of which Fig. 13-3 shows the Renold silent chain. These

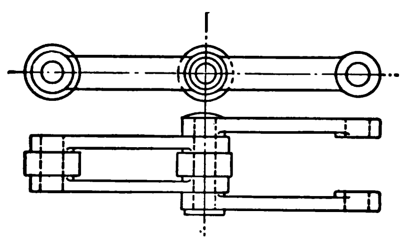


FIG. 13-2.

chains may be used on very short as well as on fairly large center distances. They give positive speed ratio and minimum pressure on bearing, as the slack side is under no tension. A speed ratio of one to seven or one to eight may easily be obtained with a single pair of sprocket wheels.

The maximum speed of block chains is about 800 ft. per minute, of roller chains 1000 ft. per minute, and the inverted tooth chain 1500 ft. per minute. While these speeds are exceeded in some cases they represent a safe limit with long life of chain and sprockets. Chains of short pitch run more smoothly and are preferable to those of long pitch. It is desirable to have the distances between centers of shafts, connected by chains, ad-

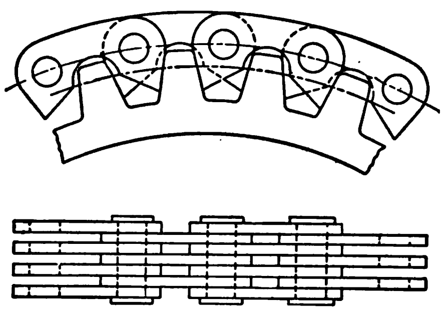


FIG. 13-3.

justable. This permits the inevitable stretch of chain to be taken up.

Sprocket wheels must be accurately cut in order to insure quiet running and long life to chain. The important dimensions are outside diameter, pitch diameter, and root diameter. For block chains

Let N = number of teeth,

b = diameter of round part of block,

C = center to center in block,

A = center to center in side links,

$$\alpha = \frac{180}{N};$$

then (Fig. 13-4),

$$\tan \beta = \frac{\sin \alpha}{\frac{C}{A} + \cos \alpha},$$

$$\text{pitch diameter} = D = \frac{A}{\sin \beta},$$

$$\text{root diameter} = D_1 = D - b.$$

For roller chain (Fig. 13–5) let

$$d = \text{diameter of roller,}$$
$$\alpha = \frac{180}{N},$$
$$p = \text{pitch of chain;}$$

then

$$D = \frac{p}{\sin \alpha},$$
$$D_1 = D - d,$$
$$D_2 = D + d;$$

for very small sprockets the outside diameter is reduced slightly.

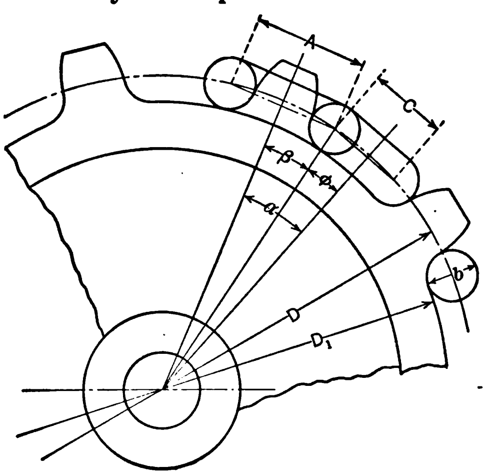


Fig. 13–4.

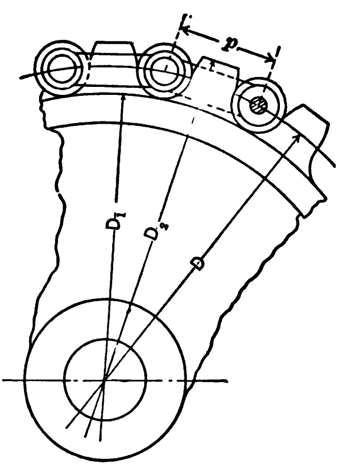


Fig. 13–5.

The chief source of trouble in chain drives is the elongation of chain. This causes it to bind on sprocket wheel and results in rapid destruction. In the inverted tooth type this trouble is obviated, the effect of elongation being simply to make the chain ride up higher on teeth of gears. The parts of transmission chain are made of high-grade steel, not so much for its greater strength, but for its wearing qualities when hardened and tempered. The tension which may be put on a chain is limited by the allowable pressure on pins or rivets. Experience has shown that this pressure should not exceed 600 or 700 lbs. per square inch or projected area.

CHAPTER XIV

PIPES AND CYLINDERS

Material and Manufacture. The metals used in the manufacture of pipes are cast iron, wrought iron, steel, copper, brass, lead, tin, etc., the first three being by far the most important in engineering work. Cast-iron pipe is used chiefly for water and other liquids as well as steam when the pressure does not exceed 100 lbs. per square inch. Wrought-iron and steel pipes are either butt or lap welded. The small size pipes up to about

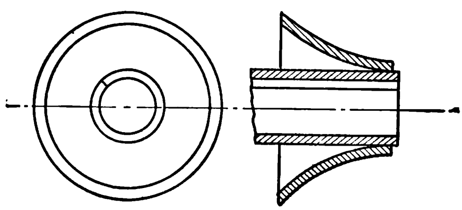


FIG. 14–1.

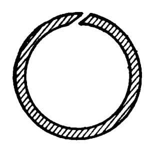


FIG. 14–2.

$2\frac{1}{2}$ ins. diameter are generally butt welded while above that size the lap welding process is employed.

Butt-welded pipe is made by drawing the plate, called skelp, through a bell-shaped die (Fig. 14–1) which bends it up and makes the weld. The plate having of course been previously heated in a gas furnace to a welding temperature. In the lap-welded tube the edges of the plate are first scarfed or bevelled and it is then bent into the shape shown in Fig. 14–2. It is then reheated to a welding temperature and passed over an arbor between a pair of rolls (Fig. 14–3) which make the weld.

A newer process makes seamless tubing. The method of manufacture is to pierce a billet of steel and force it over an arbor between two rolls. The axes of these rolls are not quite parallel, which gives the billet a rotary as well as a forward motion, thus drawing it out into a tube.

Pipes and cylinders of very large diameter are built up by riveting together steel or iron plate.

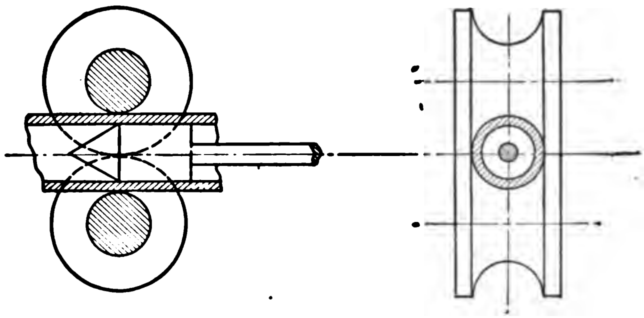


FIG. 14-3.

Strength of Pipes and Cylinders. A pipe of length L and diameter D (Fig. 14-4) is subjected to an internal fluid pressure of p lbs. per square inch. If we imagine any plane as AB passed through the axis of the pipe the component (R) of the pressure,

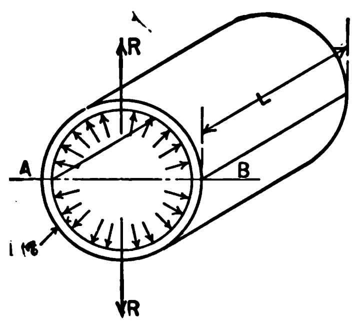


FIG. 14-4.

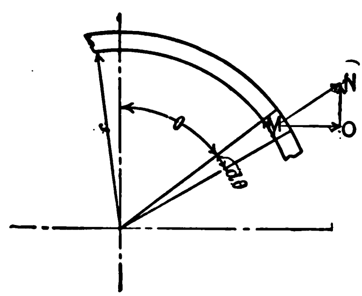


FIG. 14-5.

acting perpendicular to this plane produces tension in the sections cut by it from the pipe. Since the total tension produced must be equal to the force producing it, then if t is the thickness of pipe and s_t the stress

$$R = 2tLs_t. \quad \ldots \ldots \ldots \quad (1)$$

Since R is the sum of the vertical components of the internal pressure its value is given by the equation.

$$R = DLP; \quad \ldots \ldots \ldots \quad (2)^*$$

* See Appendix C for derivation.

then from (1) and (2),

$$DLP = 2tLs_t;$$

and therefore

$$t = \frac{Dp}{2s_t}. \qquad (3)$$

In practice this equation is modified to allow for various contingencies such as shock due to water hammer and to handling while in transportation, stresses due to method of supporting pipe, etc. This is allowed for by adding a certain amount to the thickness of pipe and equation (3) becomes

$$t = \frac{Dp}{2s_t} + k; \qquad (4)$$

k is a constant the value of which depends on the material of the pipe and on the methods of manufacture. Table 14–1 gives average values of this constant and of s_t.

In built-up cylinders and pipes the weakest portion is the joint and its efficiency will determine the thickness of plate required. For this case then if e is the efficiency of the joint expressed decimally equation (3) becomes

$$t = \frac{Dp}{2s_t e}. \qquad (5)$$

TABLE 14–1

	k In.	s_t
Cast-iron pipe	$\frac{1}{4}$	4,000
Welded-steel pipe	$\frac{1}{16}$	20,000
Seamless steel pipe	0	40,000
Copper pipe	$\frac{1}{8}$	8,000
Cast-iron engine cylinders	$\frac{1}{2}$	3,000
Cast-iron pump cylinder	$\frac{3}{8}$	3,000

The stress in a transverse section of a pipe or cylinder is one-half that in the longitudinal section. In Fig. 14–6 is shown a cylinder closed at the ends. Pass a plane AB through it perpendicular to the axis it will cut a ring section the area of which

is very nearly πDt (if D is large in comparison with t). The total stress in this section is equal to the pressure on end of cylinder, or

$$\frac{\pi}{4}D^2p = \pi Dts_t;$$

and therefore

$$s_t = \frac{Dp}{4t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

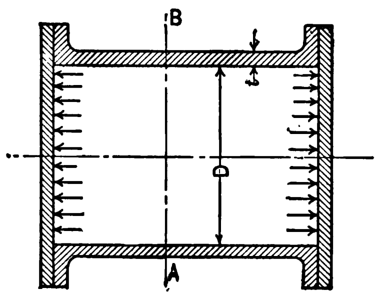


FIG. 14–6.

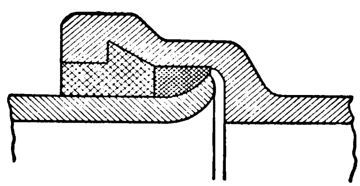


FIG. 14–7.

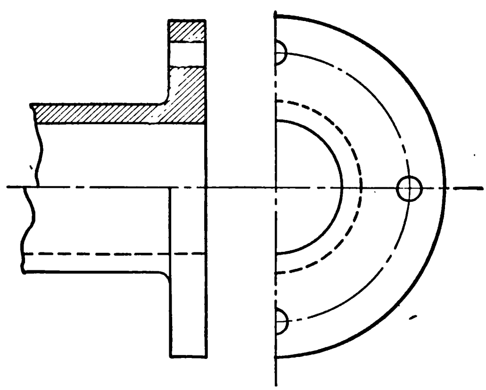


FIG. 14–8.

Pipe Joints. There are many different joints used for connecting the lengths of pipes. For cast-iron gas or water pipe, laid under ground, the bell-and-spigot joint shown in Fig. 14–7 is commonly used. To make a tight joint it is first packed with jute or hemp and then lead is poured into the remaining space. Cast-iron steam pipe is flanged as shown in Fig. 14–8. A gasket

of some soft material such as rubber, asbestos, etc., is used and the flanges are bolted together. The table below gives dimensions of cast-iron pipe and flanges as adopted by the Amer. Soc. of Mech. Engs. and the Master Steam and Hot Water Fitters Assoc.

TABLE 2

Pipe Size in Inches.	Thickness.	Diameter of Flange.	Thickness of Flange.	Diameter of Bolt Circle.	Number of Bolts.	Diameter of Bolts.
2	.409	6	$\frac{5}{8}$	$4\frac{3}{4}$	4	$\frac{5}{8}$
$2\frac{1}{2}$	.429	7	$\frac{11}{16}$	$5\frac{1}{2}$	4	$\frac{5}{8}$
3	.448	$7\frac{1}{2}$	$\frac{3}{4}$	6	4	$\frac{5}{8}$
$3\frac{1}{2}$	.466	$8\frac{1}{2}$	$\frac{13}{16}$	7	4	$\frac{5}{8}$
4	.486	9	$\frac{15}{16}$	$7\frac{1}{2}$	4	$\frac{3}{4}$
$4\frac{1}{2}$	.498	$9\frac{1}{4}$	$\frac{15}{16}$	$7\frac{3}{4}$	8	$\frac{3}{4}$
5	.525	10	$\frac{15}{16}$	$8\frac{1}{2}$	8	$\frac{3}{4}$
6	.563	11	1	$9\frac{1}{2}$	8	$\frac{3}{4}$
7	.600	$12\frac{1}{2}$	$1\frac{1}{16}$	$10\frac{3}{4}$	8	$\frac{3}{4}$
8	.639	$13\frac{1}{2}$	$1\frac{1}{8}$	$11\frac{3}{4}$	8	$\frac{3}{4}$
9	.678	15	$1\frac{1}{8}$	$13\frac{1}{4}$	12	$\frac{3}{4}$
10	.713	16	$1\frac{3}{16}$	$14\frac{1}{4}$	12	$\frac{7}{8}$
12	.790	19	$1\frac{1}{4}$	17	12	$\frac{7}{8}$
14	.864	21	$1\frac{3}{8}$	$18\frac{3}{4}$	12	1
15	.904	$22\frac{1}{4}$	$1\frac{3}{8}$	20	16	1
16	.946	$23\frac{1}{2}$	$1\frac{7}{16}$	$21\frac{1}{4}$	16	1
18	1.02	25	$1\frac{9}{16}$	$22\frac{3}{4}$	16	$1\frac{1}{8}$
20	1.09	$27\frac{1}{2}$	$1\frac{11}{16}$	25	20	$1\frac{1}{8}$
22	1.18	$29\frac{1}{2}$	$1\frac{13}{16}$	$27\frac{1}{4}$	20	$1\frac{1}{4}$
24	1.25	32	$1\frac{7}{8}$	$29\frac{1}{2}$	20	$1\frac{1}{4}$
26	1.30	$34\frac{1}{4}$	2	$31\frac{3}{4}$	24	$1\frac{1}{4}$
28	1.38	$36\frac{1}{2}$	$2\frac{1}{16}$	34	28	$1\frac{1}{4}$
30	1.48	$38\frac{3}{4}$	$2\frac{1}{8}$	36	28	$1\frac{3}{8}$
36	1.71	$45\frac{3}{4}$	$2\frac{3}{8}$	$42\frac{3}{4}$	32	$1\frac{3}{8}$
42	1.87	$52\frac{3}{4}$	$2\frac{5}{8}$	$49\frac{1}{2}$	36	$1\frac{1}{2}$
48	2.17	$59\frac{1}{2}$	$2\frac{3}{4}$	56	44	$1\frac{1}{2}$

For pipe carrying up to 200 lbs. per square inch pressure.

Wrought-iron and steel pipes find a wide application in various fields and the joints used are very numerous. For small pipe and

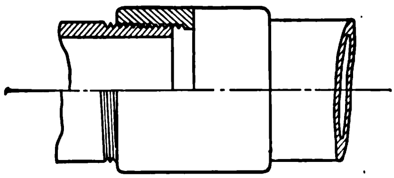


Fig. 14–9.

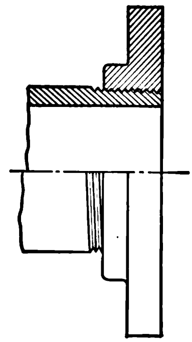


Fig. 14–10.

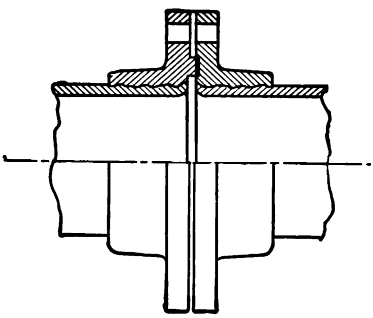


Fig. 14–11.

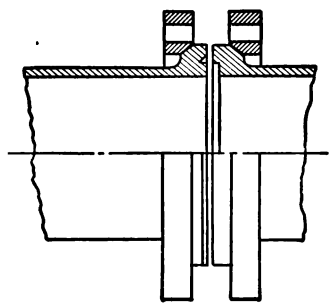


Fig. 14–12.

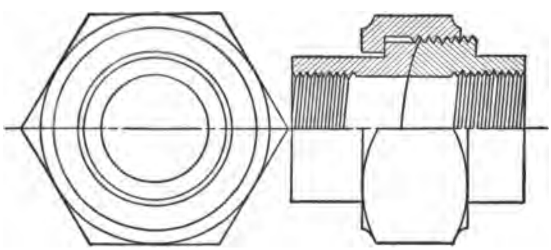


Fig. 14–13.

low pressures a coupling as in Fig. 14–9 is used. For high pressures and large pipe some form of flanged joint is used. Fig 14–10 shows a screwed flange joint. Fig. 14–11 a joint in which

the flanges are fastened to pipe by expanding it into grooves turned into the hub of flanges. To make a tight joint the flanges are provided with tongue and groove. Fig. 14–12 shows a joint with loose bolt flanges, and rings welded on, which are provided with a turned groove for a copper wire ring packing. These last three are examples of the type of joint used for high pressures and superheated steam. Fig. 14–13 is a joint used for small pipes called a union. This provides an easy method of taking pipes apart where it may be necessary occasionally.

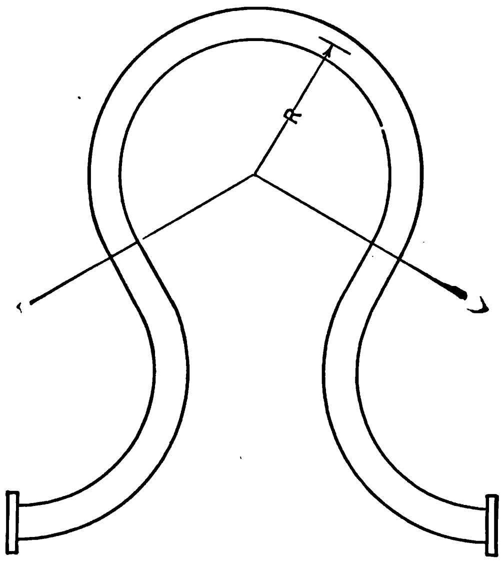


FIG. 14–14.

Expansion Joints. Long lines of pipe handling hot fluids expand to such an extent that provision must be made for this if leaky joints or breaking of pipe is to be prevented. This is particularly true with the high-pressure and highly superheated steam used in present day steam engineering practice. The expansion per 100 ft. of pipe carrying steam at 100 lbs. pressure will be about 2½ ins. With high pressure and superheat it will be about double this amount. The simplest way of providing for the necessary flexibility is by means of expansion bends, one form of which is shown in Fig. 14–14. These may be used for

pipes up to 12 ins. diameter. The radius R should not be less than five times the diameter of pipe. Fig. 14–15 shows a type of expansion joint with gland and stuffing box, perhaps the most satisfactory way of taking care of the expansion in long lines of pipe.

Support for Pipe Lines. Expansion causes motion of the entire line of pipes. Therefore, in order to have the expansion joints perform their duty, it is essential to provide fixed points at which the pipe is firmly anchored; between two such pionts an expansion joint is placed. Besides these the pipe must of course have fre-

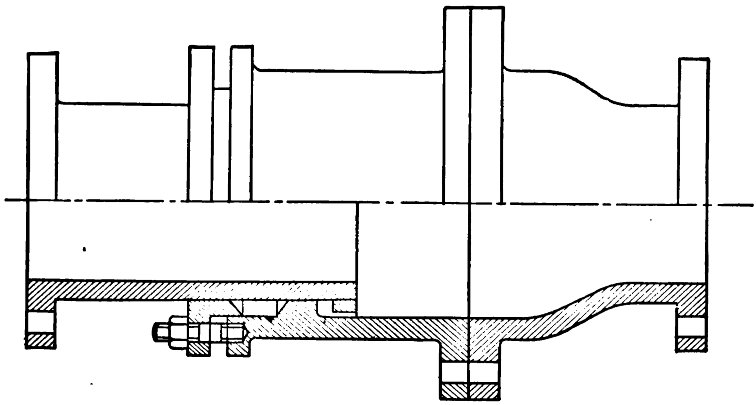


FIG. 14–15.

quent points of support to prevent undue bending stresses. Fig. 14–16 shows a roller support and bracket for large pipes. Fig. 14–17 is an example of one of the many types of overhead supports or hangers.

Thick Cylinders. For very high pressures such as are common in hydraulic presses, the cylinders are very thick and it is found that the stress is no longer uniformly distributed. It is a maximum on the inside and decreases toward the outside. Since equation (3) is based on the assumption of uniform stress it is not applicable to this case. Bach has investigated this subject experimentally and has deduced an approximate equation as follows:

Let D = outside diameter of cylinder,
d = inside diameter of cylinder;

then

$$\frac{D}{d}=\sqrt{\frac{s_t+.4p}{s_t-1.3p}};$$

but

$$D-d=2t;$$

and therefore

$$t=\frac{d}{2}\left(\sqrt{\frac{s_t+.4p}{s_t-1.3p}}-1\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

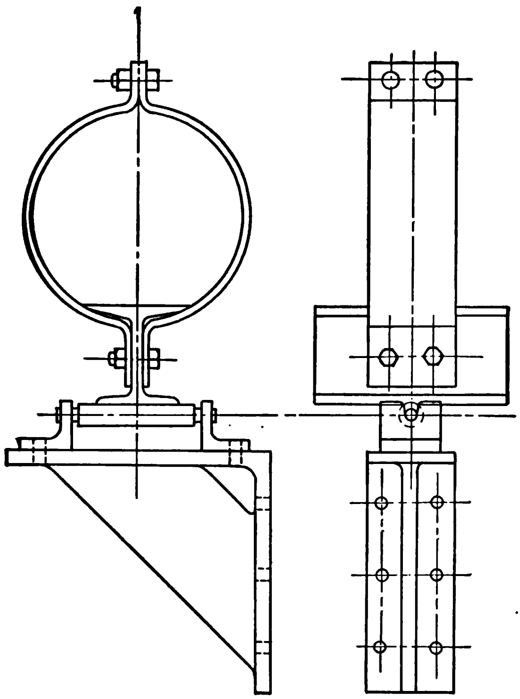


FIG. 14–16.

Size of Pipe. It is frequently necessary to determine the size of a pipe to deliver a given quantity of a fluid when the velocity of the fluid is known.

Let d = inside diameter of pipe in inches,
A = internal sectional area in square inches,
V = velocity of fluid in feet per min.,
Q = quantity of fluid to be delivered in cubic feet per minute;

then

$$Q = \frac{VA}{144};$$

and

$$A = \frac{\pi}{4} d^2;$$

$$\therefore \quad Q = \frac{\pi V d^2}{576};$$

and

$$d = \sqrt{\frac{576Q}{\pi V}}$$

$$= 13.5\sqrt{\frac{Q}{V}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

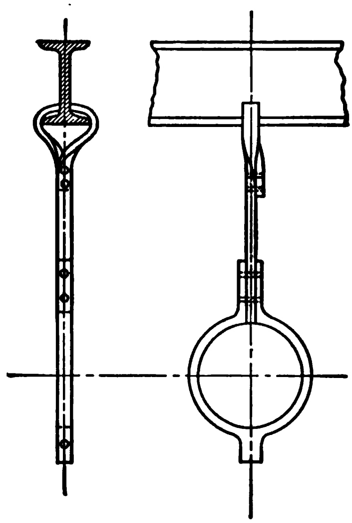


Fig. 14–17.

Example. What should be the internal diameter of a pipe which delivers the feed water to a boiler requiring a maximum of 5000 lbs. per hour, the velocity of water in pipe to be 150 ft. per min.

Since water weighs approximately 62.4 lbs. per cubic foot,

$$Q = \frac{5000}{62.4 \times 60} = 1.3 \text{ cu.ft. per min.}$$

and

$$d = 13.5\sqrt{\frac{1.3}{150}}$$
$$= 1\tfrac{1}{4} \text{ ins.}$$

PROBLEMS

√ 1. The thickness of a standard wrought iron pipe 10 ins. diameter is .366 in. This pipe is tested at factory to a pressure of 250 lbs. per square inch. Determine stress in pipe during test.

2. Find thickness of steam-engine cylinder if its diameter is 15 ins. and it operates with steam at 125 lbs. per square inch.

√ 3. Find thickness of shell for a boiler 60 ins. in diameter. The steam pressure is 100 lbs. per square inch. The longitudinal seam is a double-riveted butt joint having an efficiency of 75 per cent.

4. A hydraulic press is to exert a pressure of 200,000 lbs. with a pressure of 1000 lbs. per square inch in the cylinder. Determine thickness of cylinder assuming it to be of cast iron and allowing a tensile stress of 3000 lbs. per square inch.

√ 5. A heating boiler evaporates 1500 lbs. of water per hour, the volume of the steam being 20 cu.ft. per pound. Determine the size of the steam main so that the steam velocity will be 2000 ft. per minute.

CHAPTER XV

VALVES

Valves are machine parts to regulate the flow or the pressure of fluids in pipes and other containers. They may be divided into two general classes: (*a*) those which have no regular periodic motion and usually are operated by hand, or, in some cases, by the pressure of the contained fluid; (*b*) those having a regular periodic motion generally obtained from the machine of which they are a part, as a steam engine valve, or due to the periodically varying pressure, as in a pump.

The surface with which the valve is in contact when closed is called the seat of the valve. Another classification is according to their motion, relative to the valve seat. Thus we have (1) *swing valves*, often called flap or clack valves, which rotate about an axis parallel to the plane of the valve seat; (2) slide valves, which slide parallel to the plane of the valve seat; and (3) lift valves, which move perpendicular to the plane of the valve seat.

Types of Valves. In the following pages are shown a few examples of the more important types of valves belonging to classification (*a*) above. Fig. 15–1 represents a *globe valve.* This is used on pipes up to about 4 or 5 ins. in diameter. The valve disc is renewable, the material being copper, bronze or some softer substance, such as vulcanized fiber or rubber. Where the valve spindle passes through the top of casing is a receptacle, called stuffing box, which is filled with a packing to prevent leakage.

If the flow is in the direction indicated by arrow, then, when valve is closed, the pressure being on top of valve disc will tend to tighten it on its seat. A disadvantage, however, is that the stem cannot be repacked while under pressure.

In the *gate valve* shown in Fig. 15–2 the flow is unobstructed and this type is commonly used for the larger sizes of pipes. In the very large sizes these valves are frequently operated by

an electric motor. The hand wheel may have only a turning motion acting as a stationary nut, or it may be attached to the valve spindle and rise with it, as in the figure.

The *check valve* is used to prevent the fluid from flowing back along a pipe when the pressure is relieved. Fig. 15–3 illustrates

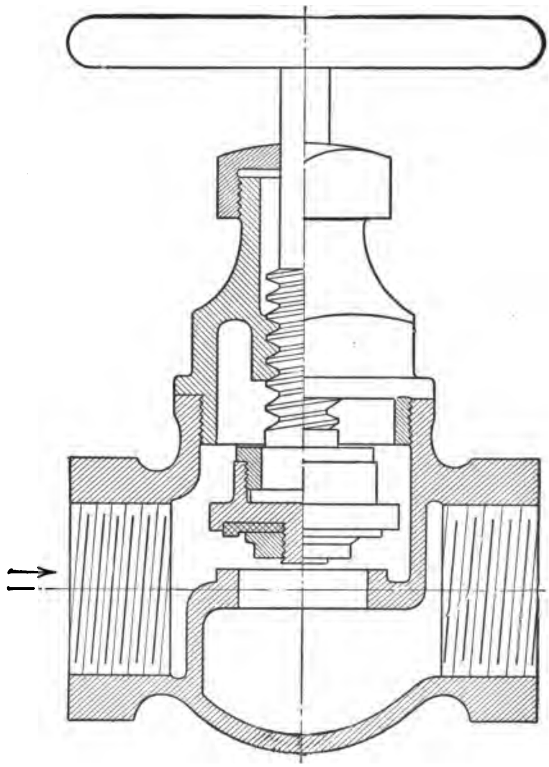


Fig. 15–1.

one example of this type. For small pipes this valve frequently takes the form of a ball resting on a spherical valve seat.

Safety Valves. Vessels containing fluids under pressure are provided with safety valves. The purpose of these is, as their name implies, to prevent the pressure rising above a safe limit; that is, they open automatically whenever the pressure reaches a predetermined limit. Their most important application is in the steam boiler. In Fig. 15–4 is shown such a valve, in which

the pressure necessary to keep it closed is obtained by means of a lever and weight. When a spring is used for this purpose it is called a pop safety valve.

Cocks. In Fig. 15–5 is shown a simple cock, consisting of a conical plug which fits into a similar shaped opening in body of

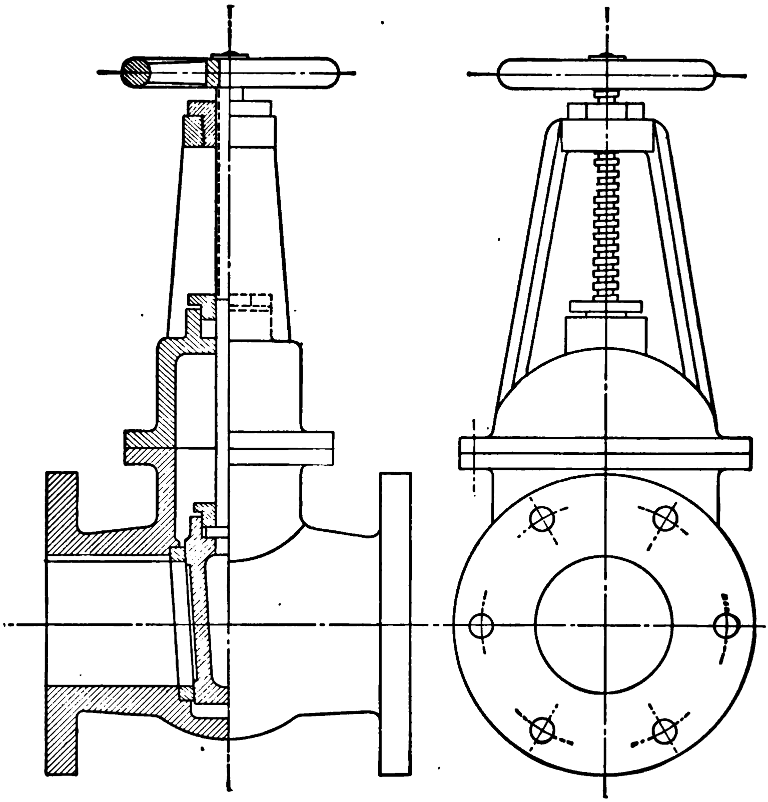


Fig. 15–2.

valve. The angle α enclosed between sides of cone varies from 8° to 15°.

The materials used for these various valve bodies are brass, cast iron and steel. Brass is used in the small sizes for both low and high pressures. Above 200° C. the strength of brass rapidly decreases, it is therefore not suitable for superheated steam.

Cast iron is used for low pressures except in the smallest sizes, where it is also employed for high pressures. For very large valves and very high pressures steel castings are the most practical.

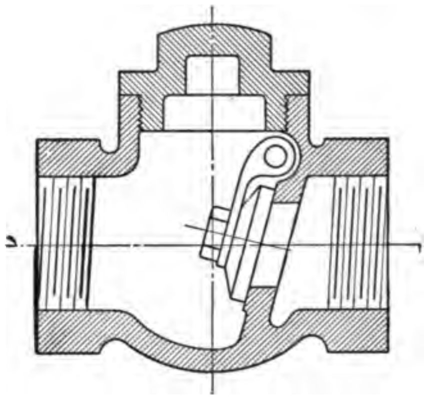


FIG. 15-3.

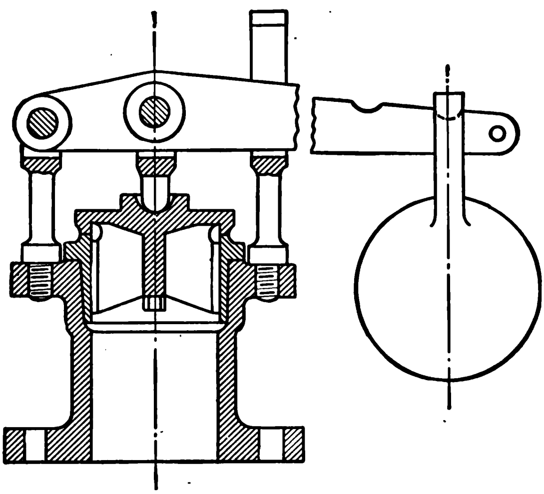


FIG. 15-4.

Lift Valves. In Fig. 15-6 are shown some of the types of lift valves commonly employed. Let h be the lift of the valve, then in order to make the valve opening equal to the area under valve we have (Fig. 15-6a).

$$\frac{\pi}{4}d^2 = \pi dh;$$

and therefore

$$h = \tfrac{1}{4}d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

This type of valve is operated by the pressure of the fluid under it as in pumps and air compressors. In order to secure quick closing of valves they are often loaded with a spring.

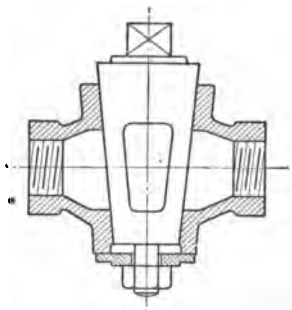
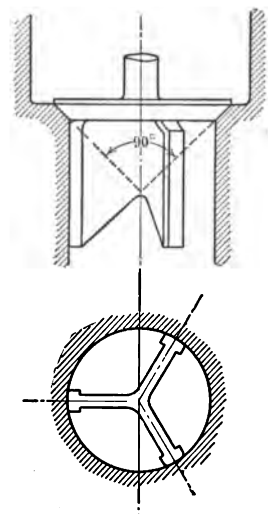


FIG. 15-5.



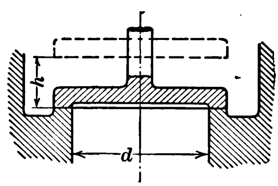


FIG. 15-6a.

FIG. 15-6b.

Let F_U = area subjected to pressure under valve,
F_A = area subjected to pressure above valve,
p_U = pressure per square inch acting on F_U,
p_A = pressure per square inch acting on F_A,
W = weight of valve,
g = acceleration of gravity,
a = acceleration of valve,
P = pressure of spring;

then for equilibrium

$$F_U p_U = F_a p_a + W + P + \frac{W}{g} a. \quad . \quad . \quad . \quad . \qquad (2)$$

That is, the valve will rise until the forces acting upwards and downwards are balanced as shown by equation (2). As soon as

the velocity of the stream through valve becomes zero, it will begin to close. But since this does not take place instantaneously, some of the liquid will flow back, furthermore, the valve will seat with an accelerated motion which causes hammering of

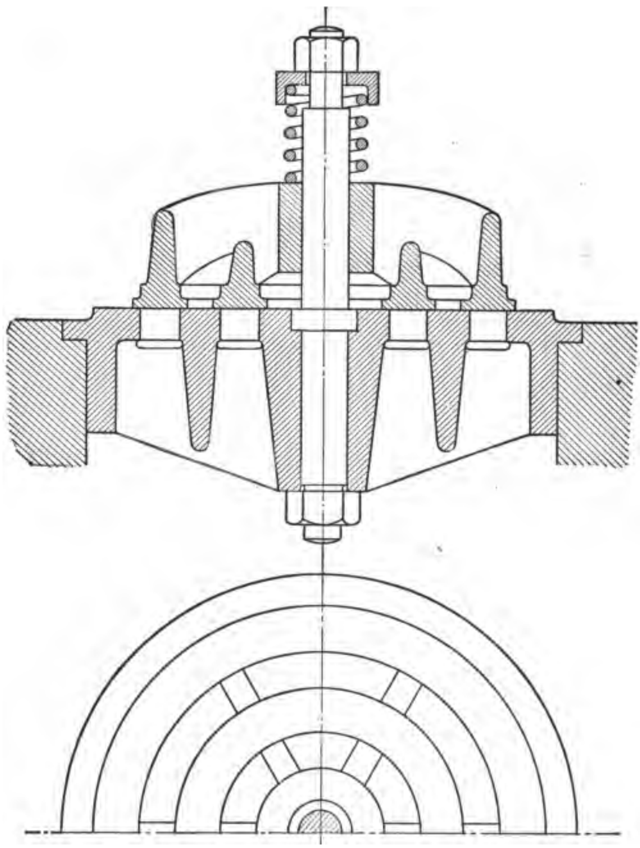


FIG. 15–7.

valve and seat; for these reasons it is desirable to make the lift as small as possible.

In very large valves the openings are a series of concentric rings, as in Fig. 15–7. This permits large valve opening with very small lift of valve. Thus if d_1 and d_2 are the outside and

inside diameters of one ring, then, assuming equal velocities through seat and valve.

$$\frac{\pi}{4}(d_1^2-d_2^2)=\frac{\pi(d_1+d_2)h}{2};$$

and

$$h=\frac{d_1-d_2}{4}=\frac{b}{2}.$$

That is, the lift of valve is equal to one-half the breadth of one ring opening; it is, however, generally increased to $.6b$ or $.7b$.

There are many forms of valves for special purposes, which can be taken up profitably only in connection with the machines of which they form a part, as they cannot be treated independently of the mechanism which operates them.

CHAPTER XVI

FLY-WHEELS

THE purpose of a fly-wheel is to maintain the speed of a machine which is doing work or receiving energy at a variable rate, between certain predetermined limits. It is a reservoir of energy, storing it up when the energy is supplied at a greater rate than the demand and giving it out when the reverse is the case. It must be understood that in order to perform its function there must be some variation in speed; that is, a fly-wheel can only store energy when its speed is increasing and give out energy when its speed is decreasing.

In such machines as punches, shears, horizontal presses, etc., fly-wheels are used, as work is done during a small portion of the cycle only. If then such a machine, without fly-wheel, were belt driven, the belt would have to be large enough to supply the entire energy consumed by the work to be done during the working portion of the cycle only, and would run idle, except for friction of machine, during the remaining portion of cycle. By the use of a fly-wheel a much smaller belt may be employed which supplies energy at approximately a constant rate throughout the cycle.

Design of Fly-wheels. To determine the weight of a fly-wheel for a machine, it is first necessary to know the amount of energy to be stored and the speed variation which may be allowed. The kinetic energy in a rotating fly-wheel, neglecting the small amount in hub and arms, is

$$K=\frac{WV^2}{2g}\text{ ft.-lbs.}$$

Where W is weight of rim in pounds, V is the mean velocity of rim in feet per second, and g is the acceleration of gravity. If

the velocity of the rim be changed from V to V_1 the kinetic energy becomes

$$K_1=\frac{WV_1^2}{2g}\text{ ft.-lbs.};$$

and therefore the energy, E, absorbed or given out during this change of speed is

$$E=K-K_1=\frac{W}{2g}(V^2-V_1^2);$$

from which we obtain

$$W=\frac{2g}{V^2-V_1^2}E. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As an illustration of the method of finding weight of a fly-wheel take the horizontal press shown in Fig. 16–1. Here A is the driv-

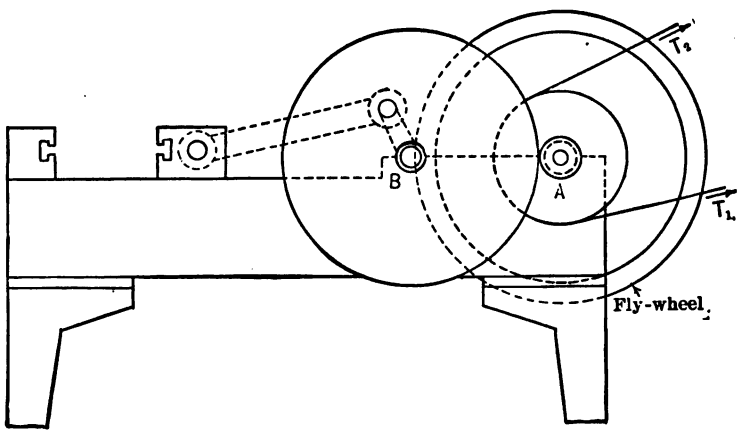


FIG. 16–1.

ing shaft and B the driven shaft. Let A make six revolutions to one of B. The work is done while B makes one-third of a revolution, the other two-thirds being idle. Let the belt speed be 1000 ft. per minute and assume the *actual* work to be done during each cycle of operations is 8000 ft.-lbs. Shaft B makes 40 R.P.M. The efficiency of machine is to be taken at 80 per cent.

The horse-power required to drive this machine is

$$H=\frac{8000\times40}{.80\times33000}=12.1.$$

The effective belt pull is

$$P = T_1 - T_2 = \frac{12.1 \times 33000}{1000}$$
$$= 400 \text{ lbs.}$$

The distance the belt travels during working portion of cycle is $\frac{1}{3} \times \frac{1000}{40} = 8\frac{1}{3}$ ft. and therefore the work done by belt during this period is $8\frac{1}{3} \times 400 = 3333$ ft.-lbs. The *total* work to be done during each cycle is $\frac{8000}{.80}$ or 10,000 ft.-lbs. The energy to be stored during idle part of cycle to be given out in working portion of cycle is

$$E = (10{,}000 - 3333).80 = 5334 \text{ ft.-lbs.}$$

Let the mean diameter of fly-wheel rim be 48 in. and the speed variation allowable 10 per cent. The fly-wheel is placed on shaft *A* and therefore makes 240 R.P.M. The mean rim speed when running at its greatest velocity is

$$V = \frac{\pi \times 48 \times 240}{12 \times 60} = 50 \text{ ft. per second.}$$

The mean rim speed when fly-wheel runs at its lowest velocity just after completing working portion of cycle is

$$v_1 = .90 \times 50 = 45 \text{ ft. per second;}$$

therefore from equation (1),

$$W = \frac{64.4}{50^2 - 45^2} 5334$$
$$= 730 \text{ lbs.}$$

Cast iron weighs .26 lb. per cubic inch, therefore the rim must contain $\frac{730}{.26} = 2800$ cu.in. The length of rim is $\pi \times 48 = 150$ ins.; then the sectional area of rim is $\frac{2800}{150} = 18.7$ sq.in., or the rim may be $3\frac{1}{2}$ by $5\frac{3}{8}$ ins. as shown in Fig. 16–2.

Very often a graphical method of design is the simplest. This is particularly the case when work is done at a variable rate and sufficient information is obtainable to draw a work diagram. In outline the process is as follows: In Fig. 16–3 let the area of *ABCD* represent the energy supplied to machine per cycle. As this area is a rectangle the energy is supplied at a constant rate. The area *AMORSD* represents the work done.

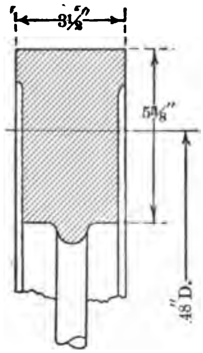


FIG. 16–2.

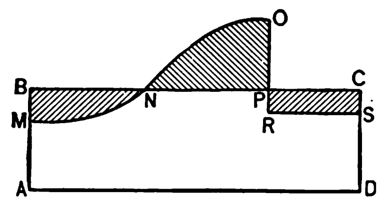


FIG. 16–3.

But the total energy supplied must equal the total work done or

$$ABCD = AMORSD.$$

Subtracting the common area *AMNPRSD* from each side of the equation we have

$$MBN + PRSC = NOP.$$

From *B* to *N* evidently the supply of energy is greater than the demand by the amount represented by the area *MBN* and this surplus is stored in the fly-wheel. From *N* to *P* the demand exceeds the supply by an amount represented by area *NOP* and the fly-wheel gives out energy; finally from *P* to *C* an amount equal to *PRSC* is again stored in fly-wheel. In other words, the diagram gives us the energy $E = NOP$, which the fly-wheel must be capable of storing up. If now its mean speed and the amount of variation permissible be known, its weight may be calculated from equation (1).

Coefficient of Fluctuation. The difference of maximum and minimum speed, in revolutions per minute, divided by the mean speed is called the coefficient of fluctuation, so that if

N = mean fly-wheel speed in R.P.M.,
N_1 = maximum fly-wheel speed in R.P.M.,
N_2 = minimum fly-wheel speed in R.P.M.

the coefficient of fluctuation is

$$\delta = \frac{N_1 - N_2}{N}.$$

Below are given usual values of δ for various kinds of machinery

	δ
Shearing and punching machines	.06–.04
Pumps	.05–.03
Flour-mill machinery	.04–.03
Paper-mill machinery and machine tools	.03–.025
Spinning machinery	.02–.01
Dynamos	.01–.003

Stresses in Fly-wheels. The stresses which occur in the rim and arms of a fly-wheel are very complicated and no solution

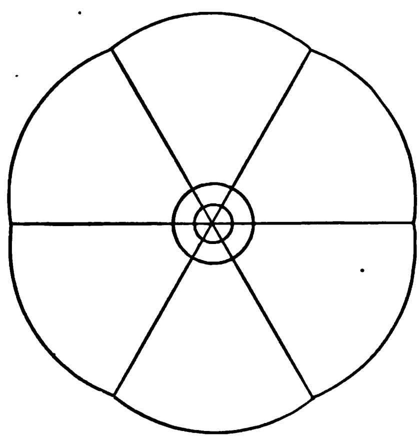


FIG. 16–4.

entirely satisfactory has been arrived at. In general the rotation of the wheel produces centrifugal forces in both the rim and the

arms, these in turn produce tensile stresses in both. But since the arms and rim are rigidly fastened together and do not stretch equal amounts the effect is to cause bending stresses in rim, as indicated in Fig. 16–4. Speed variations still further complicate the problem as the inertia forces caused thereby act tangentially and produce bending stresses in the arms.

The tensile stress in a rotating iron ring due to centrifugal force may be found exactly in the same way as was done for leather belting in Chapter IX, and the same equation holds good, that is

$$c_t = \frac{12WV^2}{g}.$$

Where

c_t = tensile stress per sq.in. due to centrifugal force,
W = weight of one cu.in. of cast iron,
V = velocity of rim in feet per second.

Substituting numerical values for W and g, we obtain

$$c_t = \frac{V^2}{10} \text{ (nearly)}. \quad \ldots \ldots \quad (2)$$

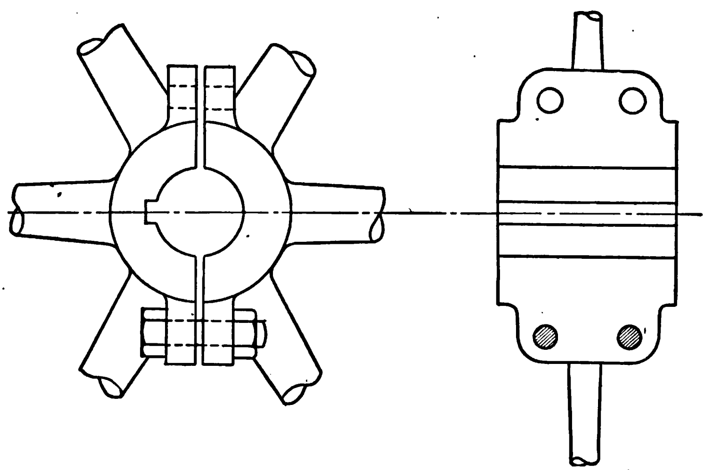


FIG. 16–5.

If we substitute the ultimate tensile stress $U_t = 20{,}000$ for c_t and solve (2) for V, we obtain the speed at which a cast-iron wheel would burst if subjected to centrifugal stresses only. The

result is $V=454$ ft. per second. It must be noted that this ultimate speed does not depend on the size or shape of the rim section. It is customary to assume that a rim speed of 100 ft. per minute is the safe limit for cast-iron wheels. For higher speeds other materials are to be preferred.

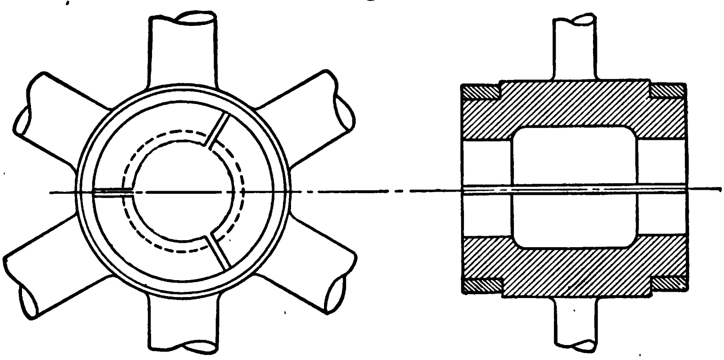


Fig. 16–6.

Construction of Fly-wheels. Small fly-wheels up to 7 or 8 ft. diameter are commonly cast in one piece. With heavy rims and hubs dangerous casting strains may result. To avoid these the hub is split into two or more parts, this enables the arms to contract freely when cooling. Figs. 16–5 and 16–6 show two

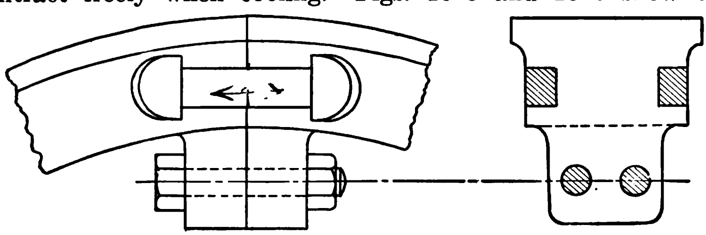


Fig. 16–7.

constructions, in the first the hub is split into two parts and bolted together, while the latter has hub split into three parts and shrink links fitted at each end. Large wheels are cast in two or more sections. Fig. 16–7 shows a wheel cast in two sections which are held together by bolts and shrink links. This joint is much weaker than the rim section besides causing heavy additional stresses due to the centrifugal force of lugs and bolts. If joint is placed in center line of arm as in Fig. 16–8 it is generally stronger than

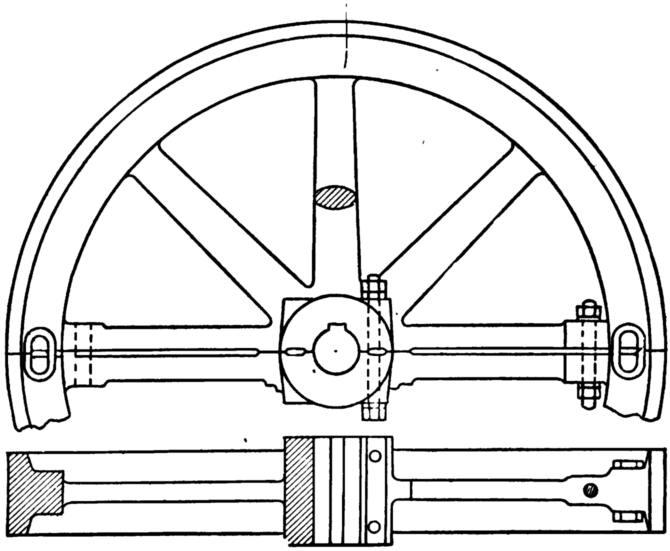


FIG. 16-8.

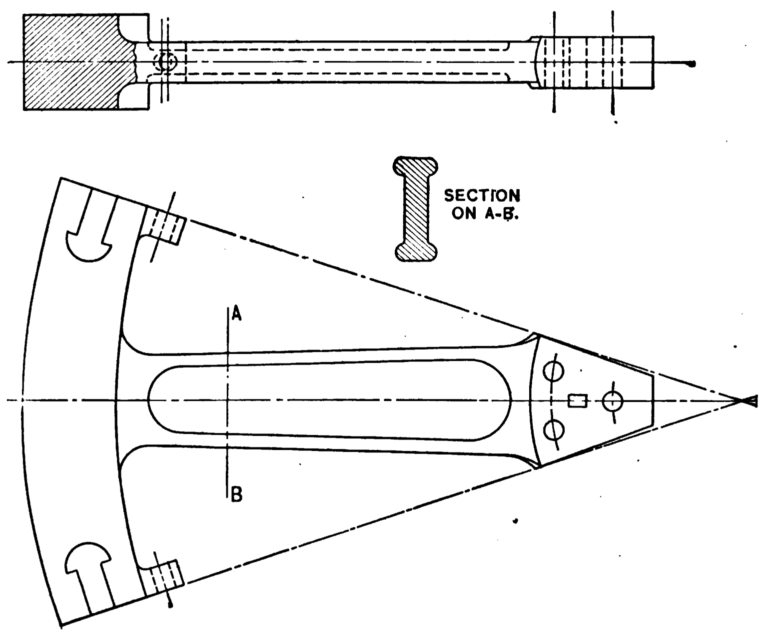


FIG. 16-9.

the first form. Fig. 16–9 illustrates a large fly-wheel in which each arm together with a portion of the rim is cast separately. The arms are bolted to a hub which is cast in two parts. The rim sections are held together by steel links. The rim is provided with barring holes for rotating wheel by means of a lever when setting valves, etc. The next illustration, Fig. 16–10 is of a fly-wheel in which the rim and arms are cast in one and are then bolted to the hub. The arms are dished, which gives them greater flexibility, and probably makes a stronger fly-wheel.

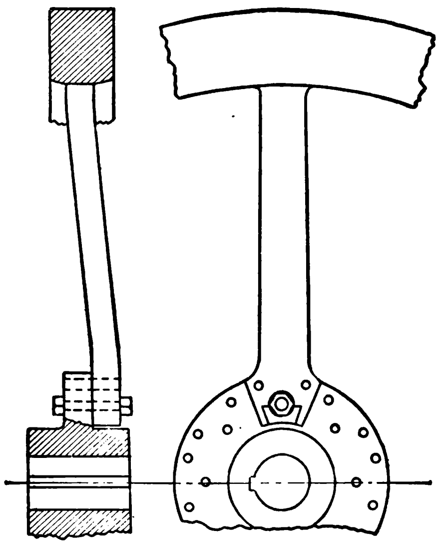


Fig. 16–10.

PROBLEMS

1. In a certain machine the mean fly-wheel diameter is 72 ins. Weight of wheel 1400 lbs. The maximum and minimum rim speeds are 175 and 160 R.P.M. respectively. Determine the change in kinetic energy of fly-wheel.

2. Determine weight of fly-wheel required for a punch-press for following conditions. Diameter of wheel (mean) 54 ins. Maximum speed 240 R.P.M., minimum speed 216 R.P.M. The machine requires 18 H.P. The complete cycle consists of eight revolutions of fly-wheel, only three of which take place during working portion of cycle. There are 28 complete cycles per minute.

3. A fly-wheel 14 ft. in diameter rotates at 125 R.P.M. The fly-wheel rim, width 16 ins. and depth =20 ins., is cast in two parts held together by two shrink links at each joint. Determine dimensions of section of these links if a stress of 20,000 lbs. per square inch is allowed.

CHAPTER XVII

CRANK-SHAFTS, CRANK-PINS, AND ECCENTRICS

Stresses in Crank Shafts. In Fig. 17–1 is shown diagrammatically the steam engine mechanism and the forces acting on the various machine elements. The steam pressure P on piston is

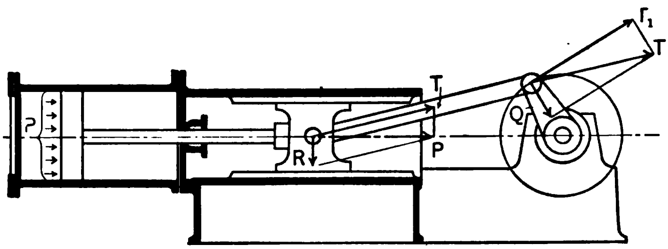


FIG. 17–1.

transmitted through the piston rod to the cross-head pin. Here it is resolved into a thrust T, in the connecting rod and a pressure R on the guides. The thrust, T, is transmitted by connecting rod to crank-pin, where it may be resolved into a tangential component T_1 and a force Q, causing either compression or tension in crank-arm.

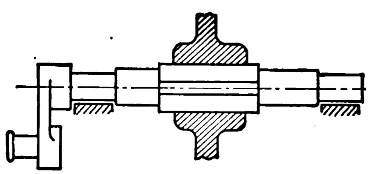


FIG. 17–2.

Taking first the crank-shaft, as shown in Fig. 17–2, having the crank at one end and carrying the fly-wheel between the two bearings, let

G = weight of fly-wheel,
P = total thrust on crank-pin (assumed equal to steam pressure on piston).

The shaft is subjected to combined twisting and bending. For convenience it may be assumed, without serious error, that this pressure on the crank-pin acts horizontally for all positions of

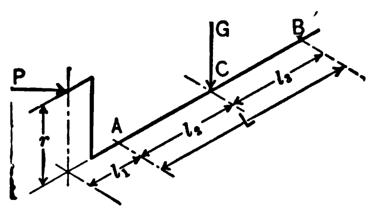


FIG. 17-3.

crank. To find the reactions at A and B find first the reactions at these supports in the horizontal plane (Fig. 17-3).

$$R_{AH}=\frac{P(l_1+L)}{L},$$

$$R_{BH}=\frac{Pl_1}{L};$$

and then in vertical plane

$$R_{AV}=\frac{Gl_3}{L},$$

$$R_{BV}=\frac{Gl_2}{L}.$$

The total reactions are

$$R_A=\sqrt{R^2{}_{AH}+R^2{}_{AV}}, \quad \ldots \ldots \quad (1)$$

$$R_B=\sqrt{R^2{}_{BH}+R^2{}_{BV}}. \quad \ldots \ldots \quad (2)$$

The bending moments at A and C may now be determined. At A

$$M_A=Pl_1. \quad \ldots \ldots \quad (3)$$

At C

$$M_C=R_Bl_3. \quad \ldots \ldots \quad (4)$$

The maximum twisting moment transmitted is

$$T=Pr.$$

The equivalent twisting moment at A is

$$T_A = M_A + \sqrt{T^2 + M_A^2}. \qquad (5)$$

At C

$$T_C = M_C + \sqrt{T^2 + M_C^2}; \qquad (6)$$

then the diameter at A is

$$d_A = 1.72\sqrt[3]{\frac{T_A}{s}}; \qquad (7)$$

and at C

$$d_C = 1.72\sqrt[3]{\frac{T_C}{s}}. \qquad (8)$$

The value of s for a mild steel shaft may be taken at from 7000 to 9000 lbs. per square inch. It is customary in designing a crank-shaft to assume tentative dimensions and use the above equations (1) to (8) to check the stress at sections A and C.

Example. The following data are taken from a small horizontal engine:

$$P = 10{,}000 \text{ lbs.},$$
$$G = 2500 \text{ lbs.},$$
$$r = 8 \text{ ins.},$$
$$d_A = 5 \text{ ins.},$$
$$d_C = 6 \text{ ins.},$$
$$l_1 = 9 \text{ ins.},$$
$$l_2 = 28 \text{ ins.},$$
$$l_3 = 24 \text{ ins.}$$

Find stress at A and C.

$$R_{AH} = \frac{10000 \times 61}{52} = 11{,}700 \text{ lbs.},$$

$$R_{BH} = \frac{10000 \times 9}{52} = 1700 \text{ lbs.}$$

$$R_{AV} = \frac{2500 \times 24}{52} = 1150 \text{ lbs.},$$

$$R_{BV} = \frac{2500 \times 28}{52} = 1350 \text{ lbs.},$$

$$R_A = \sqrt{(11700)^2 + (1150)^2} = 11{,}750 \text{ lbs.},$$

$$R_B = \sqrt{(1700)^2 + (1350)^2} = 2170 \text{ lbs.},$$

$$M_A = 9 \times 10{,}000 = 90{,}000 \text{ in.-lbs.},$$
$$M_C = 2170 \times 24 = 52{,}500 \text{ in.-lbs.}$$
$$T = 10{,}000 \times 8 = 80{,}000 \text{ in.-lbs.},$$
$$T_A = 90{,}000 + \sqrt{80000^2 + 90000^2}$$
$$= 210{,}000 \text{ in.-lbs.},$$
$$T_C = 52500 + \sqrt{80000^2 + 52500^2}$$
$$= 148{,}000 \text{ in.-lbs.}$$

Then the stress at section A is (from equation (7))

$$s = \frac{5.1 \times 210000}{125} = 8550 \text{ lbs. per square inch;}$$

and at section C

$$s = \frac{5.1 \times 148000}{216} = 3580 \text{ lbs. per square inch.}$$

Cranked-shafts. This type of crank-shaft is used in horizontal engines having the forked frame, called center-crank engine, and

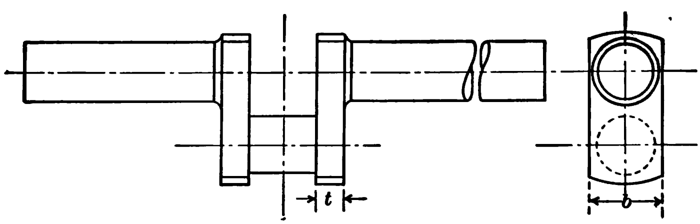


FIG. 17–4.

in nearly all vertical engines; such a shaft is shown in Fig. 17–4. In small engines the shaft is forged out solid, while for large

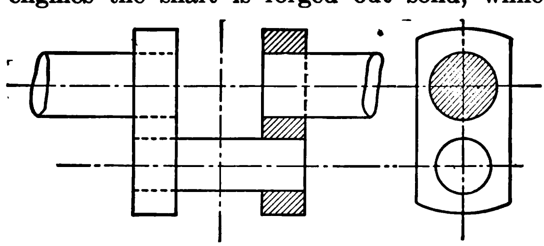


FIG. 17–5.

multiple-cylinder engines it is built up as in Fig. 17–5. Here the shaft, crank-cheeks and crank-pins are forged separately and

fastened together by means of shrink or force fits, a key being used on each crank as an additional safeguard.

Calculation of Cranked Shaft. Let Fig. 17–6 represent in outline the crank-shaft of a vertical engine. In this case the steam pressure on piston and the weight of fly-wheel act in the same direction. Then

$$R_A=\frac{Pl_2-Gl_3}{L}, \qquad (9)$$

$$R_B=\frac{Pl_1+G(L+l_3)}{L}. \qquad (10)$$

At B the bending moment is

$$M_B=Gl_3; \qquad (11)$$

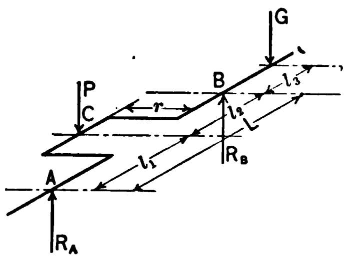


Fig. 17–6.

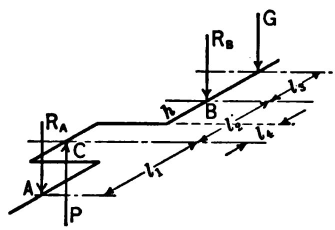


Fig. 17–7.

and the twisting moment is

$$T=Pr. \qquad (12)$$

The equivalent twisting moment is

$$T_e=M_B+\sqrt{T^2+M_B^2}; \qquad (13)$$

and the diameter of shaft at B is

$$d_B=1.72\sqrt[3]{\frac{T_e}{s}}. \qquad (14)$$

The diameter at A is usually made equal to that at B, or the bearing is designed so as to give equal bearing pressure at A and B.

The bending stress at C will be a maximum on the upward stroke as in Fig. 17–7. For this position

$$R_A=\frac{Pl_2+Gl_3}{L}; \qquad (15)$$

and the bending moment at C is

$$M_C = R_A l. \qquad (16)$$

The twisting moment is again

$$T = Pr. \qquad (17)$$

The equivalent twisting moment for this section is

$$T_e = M_C + \sqrt{T^2 + M_C^2}; \qquad (18)$$

the diameter at C is

$$d_C = 1.72\sqrt[3]{\frac{T_e}{s}}, \qquad (19)$$

or

$$s = \frac{5.1T_e}{d_C^3}. \qquad (20)$$

It is better to design this part of shaft for bearing pressure and use equation (20) as a check to keep stress within safe limits.

Taking the crank cheek, the maximum stress will be at the section h, where it joins the shaft. The bending moment at this section is

$$M_h = Pr; \qquad (21)$$

and the twisting moment is

$$T_h = R_A(l_1 + l_4) - Pl_4; \qquad (22)$$

and the equivalent bending moment is

$$M_h^1 = \tfrac{1}{2}(M_h + \sqrt{M_h^2 + T_h^2}); \qquad (23)$$

then if b is breadth and t is thickness of cheek (see Fig. 17–4)

$$M_h^1 = \tfrac{1}{6}b^2ts; \qquad (24)$$

or

$$s = \frac{6M_h^1}{b^2t}. \qquad (25)$$

In the above calculations it has been assumed that the maximum stress will occur when crank is at right angles to line of cross-head motion.

Cranks. In side-crank engines where an overhung crank-pin is used the crank is made separate from the crank-shaft to which it is fastened by a force or shrink fit with the additional safeguard of a pin key. Fig. 17–8 is an example of a forged crank.

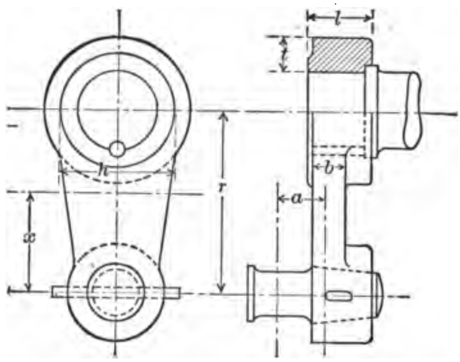


FIG. 17–8.

The material used is generally steel, sometimes wrought iron; cast iron only for small, cheaply constructed engines. In Fig. 17–9 is shown a cast-steel crank.

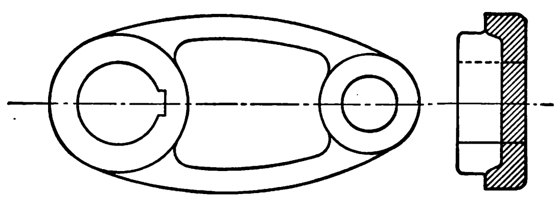


FIG. 17–9.

The stresses to which the crank-arm is subjected are combined bending and torsion. Thus in Fig. 17–8 the bending moment at any section distant x from crank pin is

$$M_x = Px;$$

the twisting moment, which is the same at all sections, is

$$T = Pa.$$

If we imagine the sides extended to center line of crank-shaft the maximum bending moment will occur at this section and is

$$M = Pr;$$

and the equivalent bending moment is

$$M_e = \tfrac{1}{2}(M + \sqrt{M^2 + T^2});$$

then

$$M_e = \tfrac{1}{6}bh^2 s; \quad \ldots \ldots \ldots \quad (26)$$

and

$$s = \frac{6M_e}{bh^2}. \quad \ldots \ldots \ldots \quad (27)$$

The stress s for wrought iron should not exceed 7000 lbs. per square inch and for steel 9000 lbs. per square inch. In designing the crank-arm the distance a is not definitely known and must be assumed. The first calculations will give simply tentative values which are used for recalculation if the assumed dimensions have been found wide of the mark. The length, l, of the boss is from $.9d$ to $1.2d$, its thickness, t, is from $.4d$ to $.5d$.

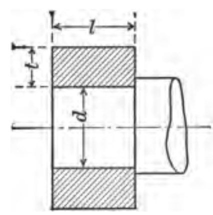


FIG. 17–10.

In calculating b and h from equations (26) we obtain the dimensions of a solid rectangle. The actual section at center of crank-shaft is shown in Fig. 17–10, and after having chosen values of t and l as given above, the section modulus of this section must be at least equal to that of the solid rectangle, or

$$\frac{bh^2}{6} \overline{<} \frac{l}{6}\left\{(d+2t)^2 - \frac{d^3}{d-2t}\right\}. \quad \ldots \quad (28)$$

EXAMPLE. In an engine having a cylinder 20 ins. diameter and taking steam at 100 lbs. per square inch the crank has the following dimensions (see Fig. 17–8): $r = 15$ ins., $a = 5\tfrac{1}{2}$ ins., $b = 3\tfrac{1}{2}$ ins., $l = 8$ ins., $t = 3\tfrac{1}{2}$ ins., $h = 12$ ins., and $d = 8$ ins. What is the maximum stress in crank? Total pressure on crank-pin is

$$P = \frac{\pi}{4} \times 20^2 \times 100 = 31{,}400 \text{ lbs.}$$

Maximum bending moment is

$$M = 31400 \times 15 = 471{,}000 \text{ in.-lbs.}$$

Twisting moment is

$$T = 31400 \times 5\tfrac{1}{2} = 172{,}500 \text{ in.-lbs.}$$

The equivalent bending moment is

$$M_e = \tfrac{1}{2}(471000 + \sqrt{471000^2 + 172500^2})$$
$$= 493000 \text{ in.-lbs.}$$

The section modulus of the dangerous section is

$$z = \tfrac{8}{6}\{(8+7) - 8^2 = 215;$$

and therefore the stress is

$$s = \frac{M_e}{z} = \frac{493000}{215} = 2290 \text{ lbs. per square inch.}$$

In high-speed engines the inertia of the reciprocating parts may increase the pressure on crank-pin considerably. This can be allowed for by increasing the factor of safety. The same thing should be done if the load on engines varies frequently and suddenly.

Crank-pins. Crank-pins are essentially journals, and the important consideration in their design is to prevent heating by keeping the bearing pressure within proper limits. The stresses to which the pin is subjected, while of secondary importance,

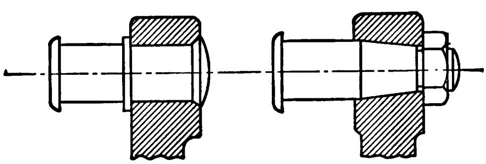


FIG. 17–11.

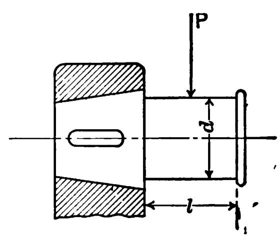


FIG. 17–12.

should in all cases be calculated to make sure they are not above a safe value for the material, which is always steel. In some cases of severe service a special high-carbon or alloy steel is used, with the bearing surface hardened and ground. In Fig. 17–11 are shown some of the various methods of fastening the pin to the crank-arm.

Calculation of Crank-pin. Let the allowable bearing pressure per square inch be q and the dimensions as shown in Fig. 17–12,

Then since the total bearing pressure must be equal to the pressure on crank-pin,

$$P = qld. \qquad (29)$$

Let the ratio of the length of pin to its diameter be denoted by n, or

$$\frac{l}{d} = n \quad \text{and} \quad l = nd;$$

substituting this value of l in (29) we obtain

$$d = \sqrt{\frac{P}{nq}}. \qquad (30)$$

The following are usual values of n: For high-speed engines $n = .8$ to 1.25; for medium-speed engines $n = 1$ to 1.4 and for low-speed engines $n = 1.25$ to 1.6. The values of q may be taken from subjoined table.

Type of Machine.	Value of q.
Crank pins of slow-speed machines with intermittent load. (Shears, punches, etc.)	2000–3000
High-speed stationary engines	400–700
Medium- and low-speed engines	600–900
Gas engines	300–600
Locomotives	1000–1800
Marine engines	400–600

The overhung crank-pin is a cantilever; the maximum bending moment is

$$M = \frac{Pl}{2}.$$

The moment of resistance of the circular section is $\frac{\pi}{32}d^3$ and therefore

$$\frac{Pl}{2} = \frac{\pi}{32} d^3 s;$$

then

$$s = 5.1 \frac{Pl}{d^3}. \qquad (31)$$

Careful lubrication is of prime importance for continuous service. In small engines an oil cup or other lubricating device is placed on the connecting rod end. In large engines the lubricant is generally fed through a hole in center of crank-pin. In Fig. 17–13 is shown the crank-shaft of a large double-acting tan-

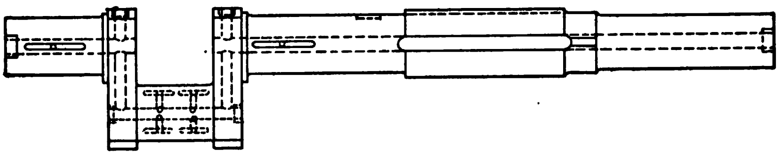


FIG. 17–13.

dem gas engine. The lubricant flows through the shaft supplying both main bearings and crank-pins.

Eccentrics. The purpose of an eccentric is to change rotary motion into reciprocating, its action being identical to that of a crank. In fact, it may be regarded as a crank mechanism in which the crank-pin has been increased in diameter until it includes the crank-shaft. Their use is limited generally to such cases

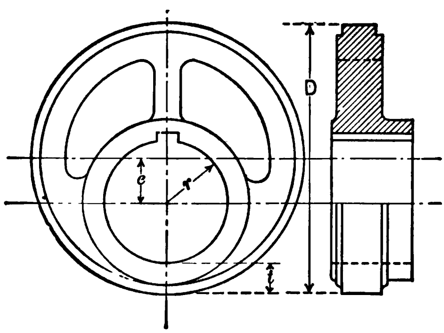


FIG. 17–14.

where the crank radius, here called eccentricity, is small and the resistance to be overcome not too heavy; their advantage being that they may readily be fastened at any point of a shaft and are a much cheaper construction than a cranked shaft.

The *eccentric sheave* consists of a disc either solid or split into two parts. The solid sheave can, of course, be used only where it can be put on over end of shaft. Fig. 17–14 shows the

construction of such a sheave. The outside diameter of sheave is

$$D=2(r+e+t), \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

where r is radius of shaft, e the eccentricity, and t the minimum radial thickness of sheave. The thickness, t, may be obtained from the empirical equation

$$t=.25r+\tfrac{1}{4} \text{ in.}$$

The material used for these sheaves is nearly always cast iron, in marine engines steel is also used.

The split sheave is shown in Fig. 17–15. Here the two parts are fastened together by studs and nuts. Instead of the nuts,

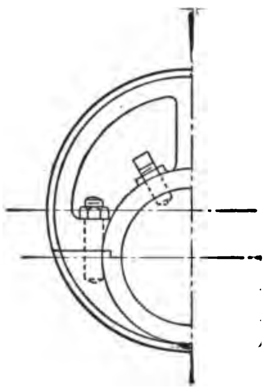


FIG. 17–15.

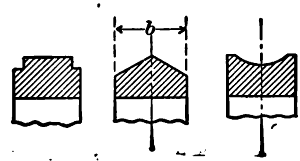


FIG. 17–16.

cotters may be used. In Fig. 17–16 the usual rim sections are shown. The width b of sheave is determined by the force P on eccentric rod and the rubbing speed of the contact surfaces. Just as in the case of crank-pins, we have (Fig. 17–12)

$$bDq=P;$$

and

$$b=\frac{P}{Dq}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Where q is again the allowable pressure per square inch of projected area, its value depends on v, the rubbing velocity in feet per minute; it may be taken at

$$q=\frac{30000}{v} \text{ to } \frac{50000}{v}.$$

In steam engine work it is often impossible to determine the force P with any degree of accuracy, and the following value of b will

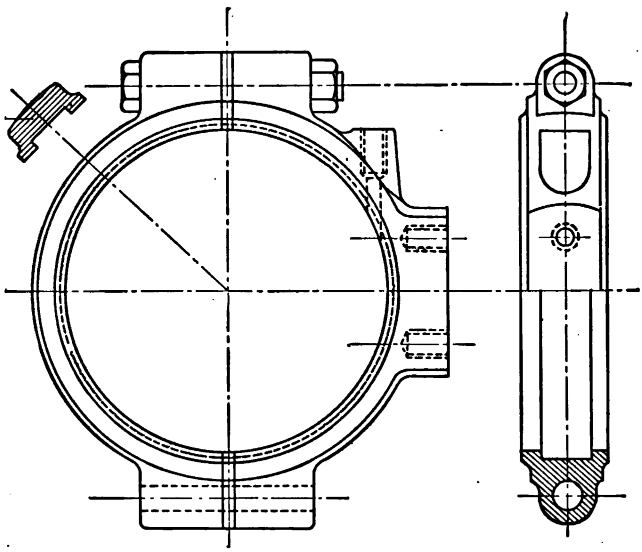


FIG. 17–17.

be found to give results agreeing closely with stationary engine practice:

$$b = .75e + 1 \text{ in.}$$

Eccentric straps are always made in two parts, fastened together by bolts, the material being generally cast iron or, for large engines, steel. In Fig. 17–17 is shown a common form of strap. For low speeds and light pressures the contact surfaces may be cast iron, otherwise the strap is lined with white metal. The eccentric rod may be screwed into strap, or it can be fastened by means of bolts as shown, or cotters. The strap is designed as a beam with a uniformly distributed load. From

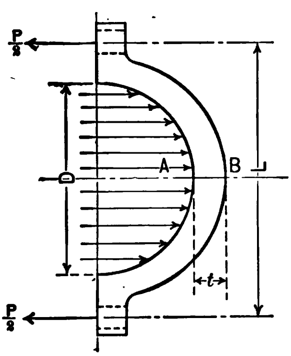


FIG. 17–18.

Fig. 17–18 the maximum bending moment at section AB is

$$M=\frac{P}{4}\left(L-\frac{D}{2}\right).$$

The section modulus of the section is

$$z=\tfrac{1}{6}bt^2,$$

assuming it to be a rectangle of breadth b and depth t, and therefore the maximum stress is

$$s=\frac{M}{z}=\frac{3P}{2}\left(\frac{L-\frac{D}{2}}{bt^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

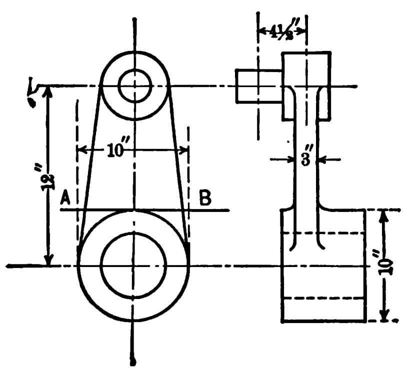
For cast iron s may be taken at 2500 to 4000 and for steel 6000 to 9000. The strap bolts are designed for tension, each sustaining a tensile force equal to $\frac{1}{2}P$.

PROBLEMS

1. A high-speed engine has a cylinder 12 ins. in diameter and uses steam at 125 lbs. per square inch. The crank-pin is of the overhung type. Make a suitable design of this pin, allowing a bearing pressure of 550 lbs. per square inch. Check design for stress.

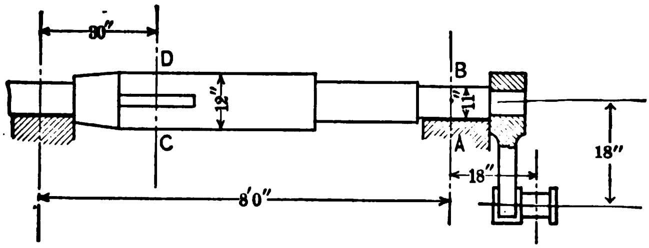
2. A 14 by 12-in. engine operating with steam at 100 lbs. per square inch has a crank-pin, overhung type, 5 ins. diameter by 4 ins. long. Determine bearing pressure and stress in pin.

3. Sketch shows the crank-arm of a 14 by 24-in. engine operating with steam at 125 lbs. per square inch. Determine the stress in section AB of crank-arm.



CHAP. XVII. Prob. 3.

4. Sketch shows the crank-shaft of a tandem compound steam engine. The maximum pressure on crank-pin is 50,000 lbs. The weight of fly-wheel is 18,000 lbs. Determine stress in shaft at sections *AB* and *CD*.



CHAP. XVII. Prob. 4.

5. The sheave of a steam engine eccentric is 10 ins. diameter. The load is 2000 lbs. Determine breadth of eccentric if engine runs at 200 R.P.M. Also design strap similar to Fig. 17–17.

CHAPTER XVIII

CONNECTING RODS, PISTON RODS, AND ECCENTRIC RODS

THE **connecting rod** is used to change the rectilinear motion of the cross-head into the rotary motion of the crank. The material used for these rods is wrought iron or steel, occasionally malleable cast iron. The rod consists of two essential parts, the ends which fit around the crank- and wrist-pins, and the shaft or rod connecting these ends.

The rod is either circular, rectangular or I-shaped in section, as shown in Fig. 18–1. The round section is commonly used

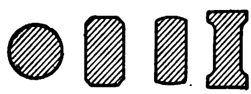


FIG. 18–1.

in long-stroke slow-speed engines, while locomotives and the short-stroke high-speed engines more frequently show the rectangular section.

Connecting rod ends show numerous variations in construction. In general there are two types, viz.: the solid, or closed end, and the open end. Fig. 18–2 shows an example of the closed type for the wrist-pin end. The boxes are of brass. They are adjusted for wear by means of a wedge or cotter. Another form of solid end is shown in Fig. 18–3. It will be noticed that the crank-pin boxes, which are cast steel, lined with babbitt metal, are adjusted for wear by a wedge which, in this case, moves at right angles to that of the previous figure. The closed end can, of course, be used only where it is possible to assemble it over the end of pin around which it fits; thus it cannot be used on center crank engines or where the crank pin is enlarged at the end to form a flange.

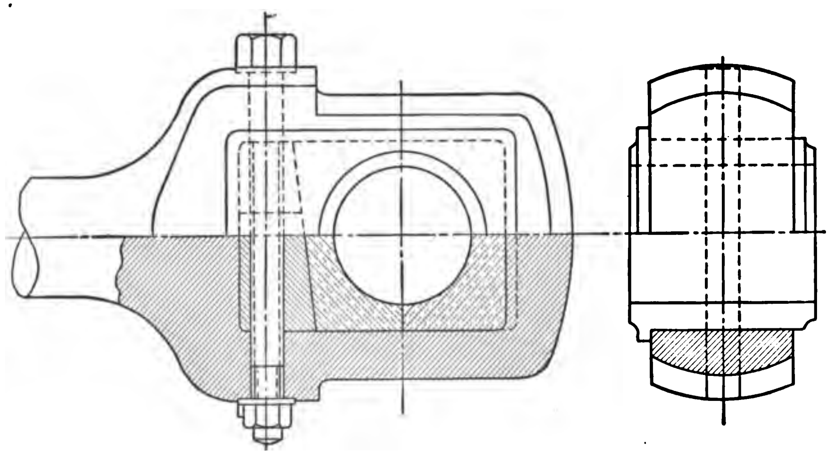


FIG. 18-2.

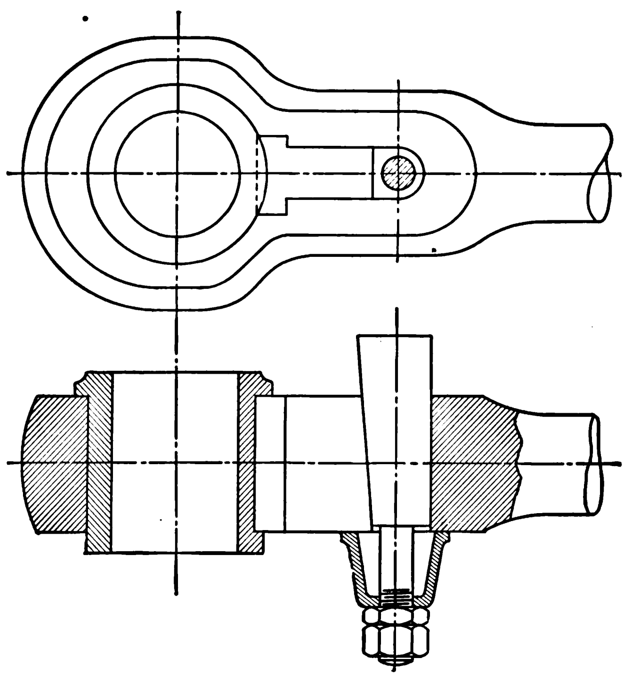


FIG. 18-3.

Fig. 18–4 is an open end called the marine type. This is a favorite design for all kinds and sizes of engines and other machinery. The cap is held by two bolts which must be of suf-

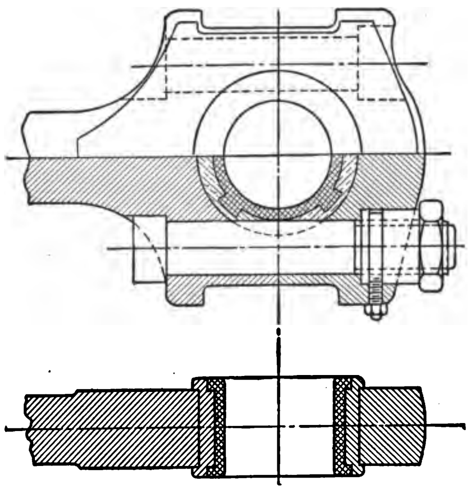


Fig. 18–4.

ficient strength to resist the entire pull due to the steam pressure upon the piston. The nuts are locked by means of set screws. Another open end, known as the strap end, is shown in Fig. 18–5. Here the boxes are held by a steel or wrought iron strap which is

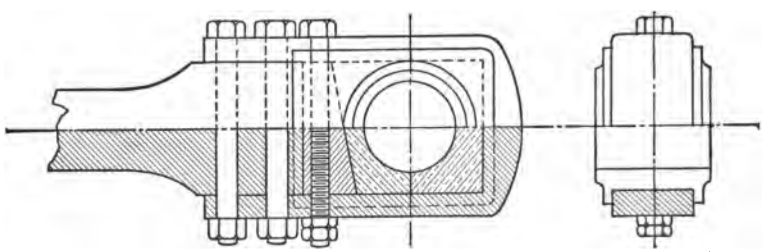


Fig. 18–5.

fastened to the stub end of connecting rod by bolts, as shown, or a cotter may replace the bolts, as in Fig. 18–6.

Connecting Rod Boxes. In the smaller sizes the boxes are either brass or bronze. For large pins they are brass or steel cast-

ing, lined with babbitt. The thickness at center of box (Fig. 18–7) may be

$$t = .15d + \tfrac{1}{8} \text{ in.}$$

At sides this is generally reduced to $t_1 = \tfrac{2}{3}t$ to $\tfrac{3}{4}t$, the length between the flanges is about

$$s = .8L.$$

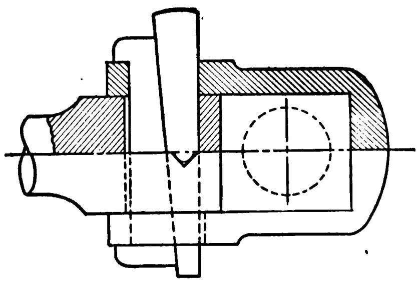


Fig. 18–6

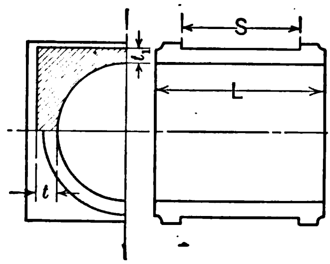


Fig. 18–7.

In taking up the wear of the boxes it should be noticed that if the adjusting wedge is on the inside brass at both ends the distance between wrist- and crank-pins will be increased, while, if placed on the outside brasses, rod is shortened. To maintain this distance constant some engine builders place one wedge on inside and one on outside brass.

Design of Rod. The rod is subjected to alternate tension and compression. While under the compression load it acts as a strut or column, and as this will produce the maximum stress in rod it must be designed as such. The length of connecting rod for stationary engines is commonly five times the crank length. Where the engine must be kept short, as in vertical marine engines, this length is about four times the crank length, in locomotives; where there is plenty of room the connecting rod is from six to eight times the crank length.

It will be convenient to assume a circular section for the rod first and obtain the diameter. An equivalent rectangular section may then be obtained, as will be shown later. In general the connecting rod may be considered a column with pin ends. If the length of rod is greater than thirty times its diameter the Euler equation given in Table 4 of Chapter I will apply. As the

rod is subjected to alternate tension and compression a high factor of safety should be used, say $F=9$ to 15.

The maximum thrust P will come on connecting rod when the crank is perpendicular to line of motion of cross-head, as in Fig.

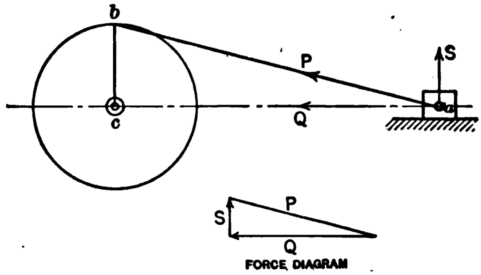


FIG. 18–8.

18–8. If Q be total pressure on piston, then from similar triangles,

$$\frac{P}{Q}=\frac{ab}{ac}=\frac{ab}{\sqrt{\overline{ab}^2-\overline{bc}^2}}$$

$$=\frac{L}{\sqrt{L^2-R^2}}.$$

Let the ratio of connecting rod length to crank length $\left(\frac{L}{R}\right)$ be n, then

$$P=Q\frac{n}{\sqrt{n^2-1}}. \qquad (1)$$

Then using the Euler column equation of Chapter I

$$P=\frac{10EI}{FL^2}.$$

Substituting for I its value $\frac{\pi}{64}d^4$ we obtain

$$P=\frac{Ed^4}{2FL^2}. \qquad (2)$$

In using this equation it is best to assume a tentative value of d and then solve for P. If the first trial solution gives a value

of P too low or considerably higher than the actual thrust on rod, other assumptions for d are made until a satisfactory result is obtained. With a little practice it requires no more than two trials.

As generally the ratio of length of connecting rod to diameter is less than 30 it will be found that the Gordon-Rankine equation is more serviceable. This for a circular section is

$$P=\frac{s_cA}{1+4k\frac{L^2}{\frac{I}{A}}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

here A is the sectional area of the rod and k is a constant whose value is $\frac{1}{3000}$ for steel or wrought iron. If we substitute this value equation (3) becomes

$$P=\frac{s_cA}{1+\frac{L^2}{750d^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

EXAMPLE. Find the diameter of a round connecting rod, the total thrust on which is 36,000 lbs.; the rod is 60 ins. long.

Make a tentative solution by assuming a diameter and solve for the safe thrust P. Assume $d=2$ ins. and for a steel rod $s=8000$ lbs., then from equation (4),

$$P=\frac{8000\times\frac{\pi}{4}\times4}{1+\frac{3600}{750\times4}}=11{,}400 \text{ lbs.};$$

this value is much too low, therefore assume $d=3$, then

$$P=\frac{8000\times\frac{\pi}{4}\times9}{1+\frac{3600}{750\times9}}=37{,}000 \text{ lbs.}$$

This is satisfactory as the error is on the side of safety.

Rectangular Rods. In case a rectangular rod is desired the diameter of a round rod is first calculated, then if t is the thick-

ness of rod (Fig. 18–9), and b is the breadth at middle, let the ratio of $\frac{b}{t}$ be denoted by n. The usual value of n ranges from 1.5 to 3. The thickness can be found from the diameter of round rod by means of the equation

$$t = \frac{.89d}{\sqrt[4]{n}}. \qquad (5)$$

The following table gives values of t for various values of n.

$n=$	1.5	1.75	2.00	2.25	2.50	2.75	3.00
$t=$	$.79d$	$.76d$	$.74d$	$.72d$	$.71d$	$.69d$	$.67d$

This type of connecting rod, when used on high-speed stationary engines, is made to taper from wrist-pin end to crank-pin end, being largest at the latter. This is in order to take care of the bending stresses acting in the plane of motion at right angles to the axis of rod. The breadth, b, at cross-head end may be about $.8b$ and at crank-end $1.2b$.

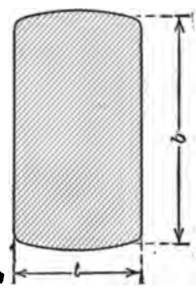


FIG. 18–9.

Connecting-rod Ends. The sides are in tension and the sectional area should be such as to keep the stress s_t within safe limits. The factor of safety should not be less than 8, as the load varies from zero to a maximum in each revolution

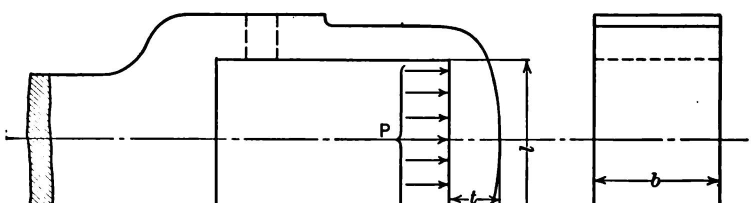


FIG. 18–10.

of engine. Since there are two areas $b \times t_1$ (Fig. 18–10) to carry the load, we have

$$2bt_1s_t = P;$$

and

$$t_1 = \frac{P}{2bs_t}. \qquad (6)$$

The cross-piece connecting the sides is regarded as a beam loaded with a uniformly distributed load. From Fig. 18–10 we have the bending moment at center

$$M=\frac{Pl}{8};$$

and

$$\frac{Pl}{8}=sz=s\tfrac{1}{6}bt^2;$$

therefore

$$t=.865\sqrt{\frac{Pl}{sb}}. \qquad (7)$$

In the marine end the cap is designed as above. The bolts are in tension and sustain one-half of the total load, P, each. The stress in bolts not to exceed 6000 lbs. per square inch for mild steel.

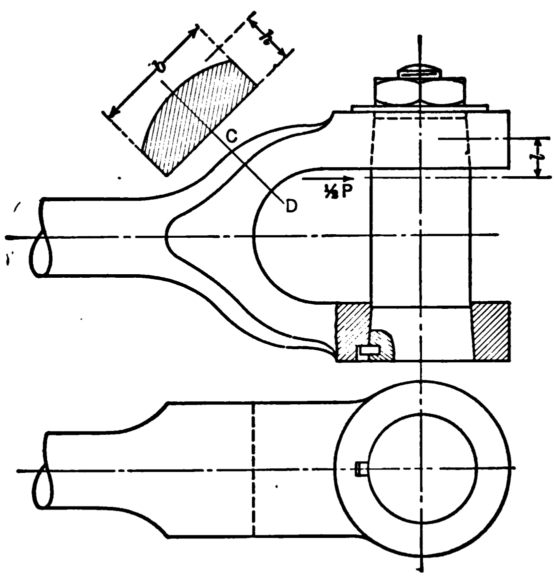


Fig. 18–11.

The forked end (Fig. 18–11) has its dangerous section at CD. This section is subjected to a direct tension

$$s_t=\frac{P}{2bh};$$

and a bending stress

$$s_b=\frac{M}{z}=\frac{\frac{P}{2}l}{\frac{1}{6}bh^2}$$

$$=\frac{3Pl}{bh^2};$$

and therefore the total stress is

$$s=s_t+s_b=P\left(\frac{1}{2bh}+\frac{3l}{bh^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Piston Rods. These rods transmit the steam pressure to the cross-head. For small engines the rod is screwed into position and riveted over, as shown in Fig. 18–12. A better method is to use a nut, as in Fig. 18–13 or Fig. 18–14.

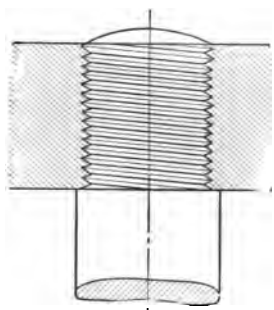


FIG. 18–12.

The piston rod is designed as a column with ends guided and therefore equation (4) will also apply here. The stress s_c may be 4000 to 6000 lbs. per square inch. With very heavy pistons the rod is also subjected to a bending stress. This may be allowed for by using a low value of s_c. If we assume $s_c=5000$ and remember that the load on piston rod is

$$Q=\frac{\pi}{4}D^2p,$$

and call the ratio $\frac{\text{length of rod } (L)}{\text{diameter of rod } (d)}=n$. then from equation (4)

$$\frac{\pi}{4}D^2p=\frac{5000\times\frac{\pi}{4}d^2}{1+\frac{n^2}{750}};$$

and

$$d=\sqrt{\frac{1+\frac{n^2}{750}}{5000}}D\sqrt{p}; \quad . \quad . \quad . \quad . \quad . \quad (9)$$

for any value of n we may write this equation

$$d = KD\sqrt{p}. \quad \ldots \ldots \ldots \quad (10)$$

The following table gives values of K for various values of n:

$n=$	8	10	12	14	16	20	25
$K=$	.0148	.0150	.0154	.0159	.0164	.0175	.0191

EXAMPLE. An engine has a cylinder 18 ins. diameter and 18-in. stroke, the steam pressure is 125 lbs. per square inch. Length of piston rod is 30 ins. Find diameter of rod.

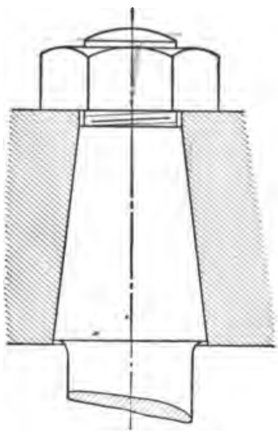


FIG. 18-13.

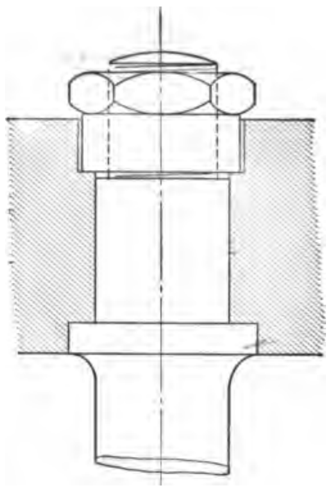


FIG. 18-14.

Assume a trial value of $n=14$ say, then $d=\frac{L}{n}=2.14$ ins. Now check by equation (10), then

$$d = .0159 \times 18 \times \sqrt{125}$$
$$= 3.20.$$

This gives a value of

$$n = \frac{30}{3.20} = 9.28.$$

Therefore our assumed value is too high; taking a second trial value of $n=10$ we obtain

$$d = \tfrac{30}{10} = 3 \text{ ins.};$$

and checking by equation (10)

$$d = .015 \times 18 \times \sqrt{125} = 3.02 \text{ ins.}$$

This is sufficiently close to our assumed value and it may therefore be taken as the diameter of piston rod.

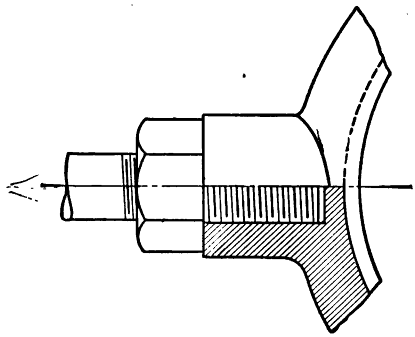


FIG. 18-15.

Eccentric Rods. These are usually round except in high-speed engines, where a rectangular section is frequently used. The material is wrought iron or steel. If the thrust on rod is

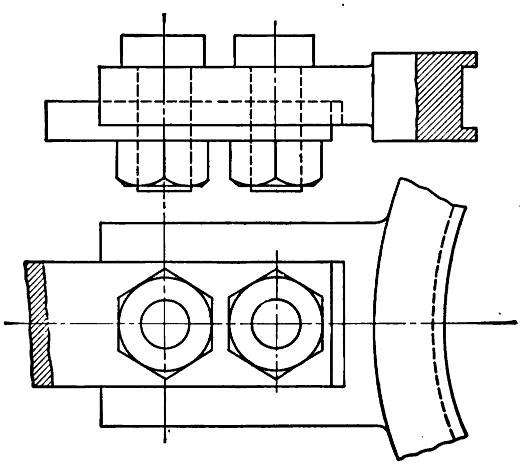


FIG. 18-16.

known they may be designed in the same manner as connecting rods. The calculated dimensions may be taken as those at center

of rod. Figs. 18–15 and 18–16 show two of the commonest types of eccentric rod ends.

PROBLEMS

1. Determine the thrust or the connecting rod of a 15 by 16-in. steam engine. The steam pressure is 160 lbs. per square inch and the length of connecting rod is 32 ins.

2. The total thrust on a round connecting rod is 50,000 lbs. Length of rod is 72 ins. Find diameter of rod if the stress is 7500 lbs. per square inch.

3. The engine in problem 1 has a rectangular connecting rod. The ratio of $b{:}t$ is 2.5. Find these dimensions if the safe stress is 9000 lbs. per square inch.

4. In a marine engine the low-pressure cylinder is 60 ins. in diameter; the maximum pressure being 40 lbs. per square inch. The piston rod is of nickel-steel. Allowing a stress of 10,000 lbs. per square inch find its diameter; the length of rod is 72 ins.

5. Design the crank-pin end of the connecting rod in problem 2. Design to be similar to Fig. 18–5. State clearly all assumptions and make your calculations complete.

CHAPTER XIX

PISTONS, CROSS-HEADS AND STUFFING BOXES

PISTONS are used in engines, pumps, air compressors and other machines. Their purpose is to take up, or to produce the pressure of fluids enclosed in cylinders. To do this they must have a fluid-tight fit and at the same time move with as little friction as possible. This is accomplished by means of packing rings, as in Fig. 19–1. Pistons which carry valves that permit the fluid to pass through them from one side of piston to the other are called buckets (Fig. 19–2). If the fluid pressure acts on one side

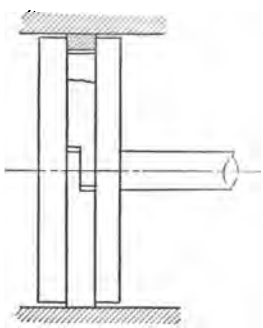


FIG. 19–1.

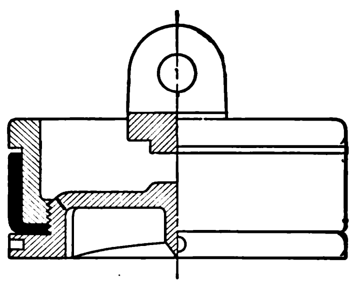


FIG. 19–2.

of the piston only and the packing is placed in the cylinder, as in Fig. 19–3, the piston is called a plunger.

The material most commonly used for pistons is cast iron. In pumps where the water to be pumped has a corrosive action brass or bronze are used. Very large pistons, especially of marine engines, are steel castings, in order to reduce their weight to a minimum.

Engine Pistons. Pistons up to about 18 ins. diameter are usually of the hollow cast-iron type shown in Fig. 19–4. For ordinary pressures the piston is provided with two cast-iron pack-

ing rings. For very high pressures three or more rings may be used. In small sizes the internal radial ribs are often omitted.

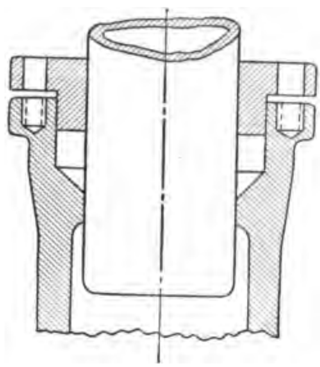


FIG. 19-3

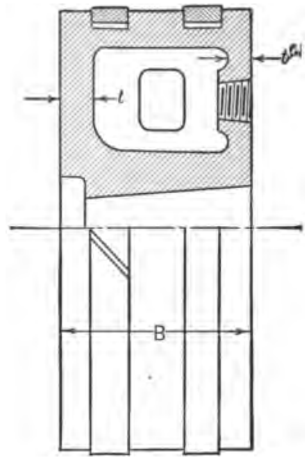


FIG. 19-4.

The purpose of the openings is to remove the core after casting. The proportions may be as follows:

$$B=\sqrt{D} \quad \text{to} \quad 1.3\sqrt{D};$$

$$t=\tfrac{1}{6}B,$$

$$t'=\tfrac{4}{3}t.$$

Large pistons are usually of the built-up type. That is, they consist of several parts bolted together. Figs. 19-5 and 19-6 show two forms of this type of piston. It will be noticed that in the built-up pistons it is not necessary to expand the packing rings so that they can be pulled over the piston castings as is the case in the solid pistons. Fig. 19-7 shows a steel piston of the type commonly used in marine engines.

In the single-acting gas engine the piston rod and cross-head are omitted, and the side thrust of connecting rod is transmitted to cylinder walls through the piston. For this reason the piston is made very long as shown in Fig. 19-8. The length L of piston may be from $L=1.25D$ to $2D$, the larger value being

used for small engines. The number of packing rings is generally from 4 to 8.

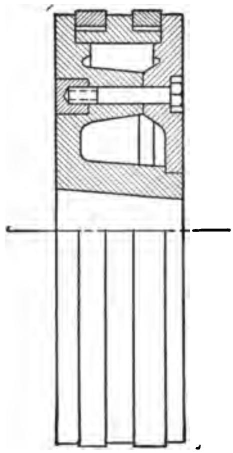


FIG. 19-5.

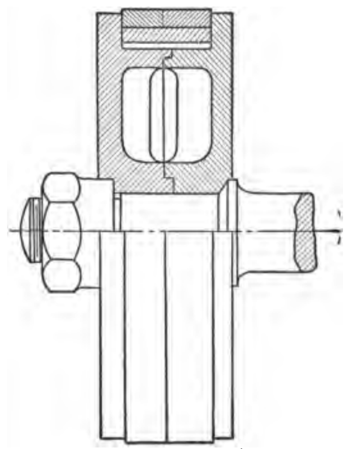


FIG. 19-6.

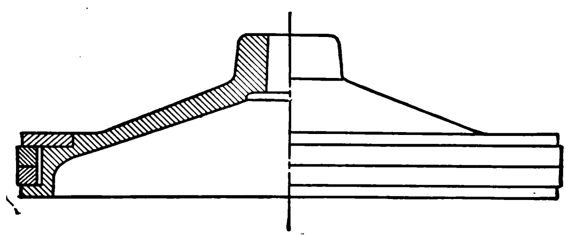


FIG. 19-7.

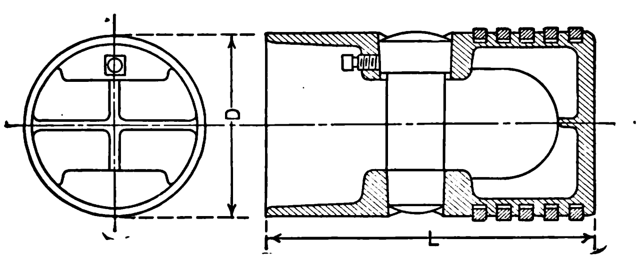


FIG. 19-8.

Piston Rings. Cast iron has been found to be the best material for packing rings. These rings are of two general types, viz.: those in which the pressure against the cylinder wall is due to

elastic forces within and those in which springs or other means are used for this purpose.

Several forms of the first type are shown in Fig. 19–9. These rings are made so that their diameter D is from 1.08 to 1.12 times the diameter of the cylinder. A piece is then cut out and when ring is placed in cylinder the ends are sprung together. The elastic stresses thus produced press the ring against the cylinder wall and make a steam-tight fit. This pressure between rings and cylinder is from 2 to 5 lbs. per square inch of contact surface. The joint shown at *a* in the figure permits some leakage of the fluid under pressure, but on account of its simplicity is often used in small engines. The joints at *b* and *c* to a large extent prevent this leakage. The following table gives good proportions for this type of piston ring:

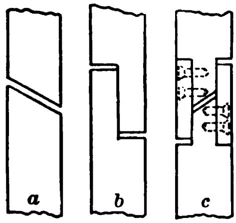


FIG. 19–9.

PISTON RINGS

Diameter.	Width.	Thickness.	Diameter.	Width.	Thickness.
D	W	t	D	W	t
4	$\frac{1}{4}$	$\frac{3}{16}$	12	$\frac{1}{2}$	$\frac{3}{8}$
6	$\frac{5}{16}$	$\frac{3}{16}$	15	$\frac{9}{16}$	$\frac{3}{8}$
8	$\frac{3}{8}$	$\frac{1}{4}$	18	$\frac{5}{8}$	$\frac{7}{16}$
10	$\frac{7}{16}$	$\frac{5}{16}$	21	$\frac{3}{4}$	$\frac{1}{2}$
			24	1	$\frac{5}{8}$

When the rings are placed in piston the joints are staggered, the rings being prevented from rotating by a pin or screw.

Cross-heads. The cross-head carries the pin called wrist-pin or gudgeon-pin and connects the piston rod and the connecting rod. It has a reciprocating straight-line motion sliding on surface formed on the *guides*. There are a great variety of constructions, the materials being cast iron, steel or wrought iron.

Fig. 19–10 shows a simple form of cross-head for a single guide, the sliding surfaces being planes. The piston rod is cottered into the hub of cross-head. In Fig. 19–11 is shown a cross-head with two cylindrical sliding surfaces formed on separate

parts called slippers. The slippers may be of cast iron, brass or cast iron with babbitted wearing surfaces. Another form is shown in Fig. 19–12. Here the slippers are arranged so that the

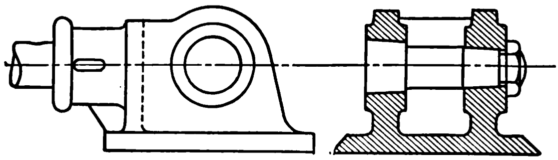


FIG. 19–10.

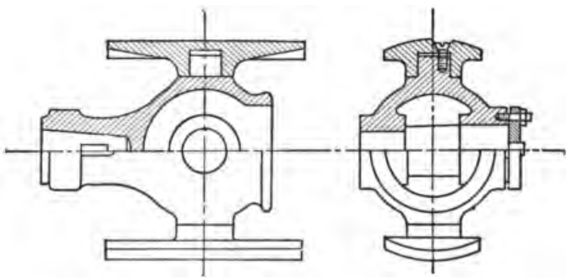


FIG. 19–11.

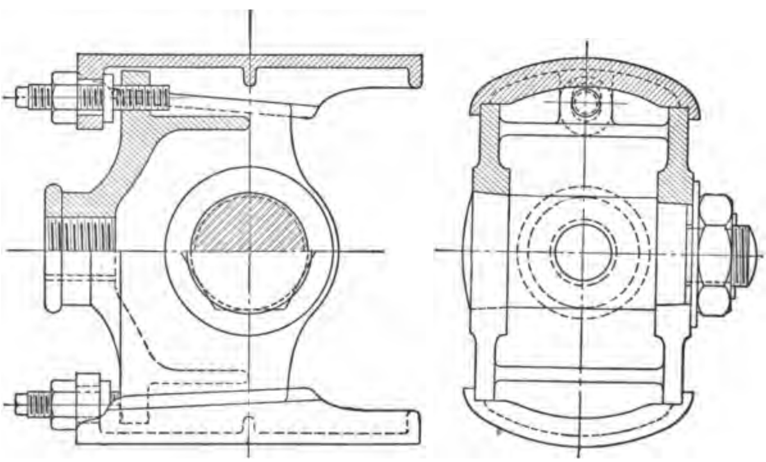


FIG. 19–12.

wear on sliding surfaces may be taken up. This type is very common in American stationary engine practice. Still another form of cross-head is shown in Fig. 19–13. Here the wrist-pin

projects on each side of cross-head and therefore a forked connecting rod is to be used.

Design. The cross-head is designed for bearing pressure. The maximum pressure will come on cross-head when crank has rotated through 90° from its dead center position, as in Fig. 19–14. If L is length of connecting rod and R length of crank, then, by finding the components of the steam pressure (Q) on piston along the

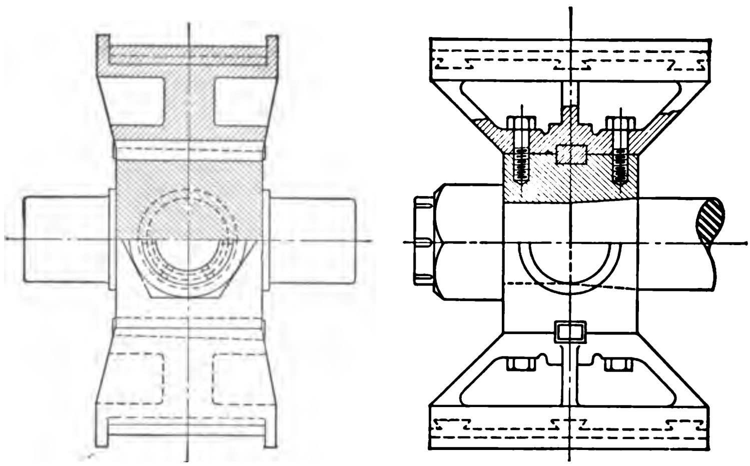


FIG. 19–13.

connecting rod (P) and perpendicular to the guides (G) we have from similar triangles

$$\frac{G}{Q}=\frac{cb}{ac}=\frac{R}{\sqrt{L^2-R^2}}. \quad \ldots \ldots \quad (1)$$

Let the ratio of connecting rod length to crank length be denoted by n, that is

$$\frac{L}{R}=n;$$

then from equation (1)

$$G=Q\frac{1}{\sqrt{n^2-1}}. \quad \ldots \ldots \ldots \quad (2)$$

Let p = the allowable bearing pressure in pounds per square inch,

a = length of bearing surface in inches,

b = breadth of bearing surface in inches;

then

$$G=abp. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The bearing pressure p ranges from 25 to 60 lbs., the lower values being used for engines having a high piston velocity.

EXAMPLE. What should be the area of the cross-head slipper for an engine using steam at 125 lbs. per square inch. The cylinder diameter is 20 ins. and the stroke 40 ins. The speed is 102 R.P.M. The connecting rod has a length of five cranks. Total steam pressure on piston is

$$Q=\frac{\pi}{4}\times\overline{20}^2\times125=39{,}500;$$

the pressure on guides is

$$G=39500\frac{1}{\sqrt{25-1}}=8000 \text{ lbs.}$$

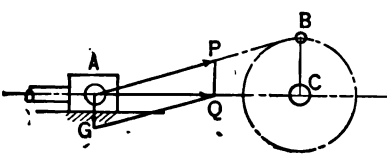


FIG. 19–14.

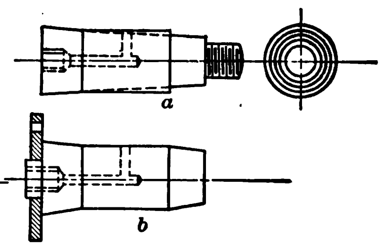


FIG. 19–15.

The mean piston speed of this engine is 680 ft. per minute. This is an average speed for medium-sized engines and we may take the allowable bearing pressure at 40 lbs. per square inch, then

$$8000=40ab;$$

and the bearing area of each slipper is

$$ab=\frac{8000}{40}=200 \text{ sq.in.};$$

or length, a, may be 18 ins. and breadth, b, about 11 ins.

Wrist-pin. This is designed for bearing pressure and checked for stress. In Fig. 19–15 are shown the usual types of wrist- or cross-head pins, which are supported at both ends. They have a taper fit in the body of cross-head and are further prevented from

rotating by a pin or key. In *a* the pin is pulled in and fastened by means of a nut at small end of taper. In *b* a plate at large end of taper presses and holds pin in position. The hole through center of pin is for applying the lubricant to the bearing surface.

The allowable bearing pressure ranges from 700 to 1400 lbs. per square inch. The method of calculating the diameter of cross-head pin is identical with that of crank-pin in Chapter XVII. Using the same symbols we obtain the same equation

$$d=\sqrt{\frac{P}{nq}}. \qquad \qquad (4)$$

The value of n for cross-head pins is 1.15 to 1.75. Having determined diameter and length according to equation (4) it is neces-

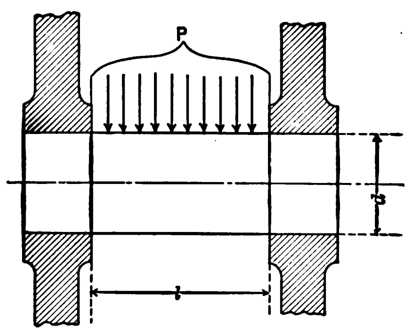


FIG. 19–16.

sary to check design for stress. Assuming the load to be uniformly distributed over pin (Fig. 19–16) the maximum bending moment is

$$M=\frac{Pl}{8};$$

and therefore

$$\frac{Pl}{8}=sz=s\,\frac{\pi}{32}\,d^3;$$

then

$$s=\frac{4Pl}{\pi d^3}=1.27\,\frac{Pl}{d^3}. \qquad \qquad (5)$$

The stress s should not exceed 5000 lbs. per square inch.

EXAMPLE. Design the wrist-pin for the engine of previous example.

Total thrust on wrist-pin is

$$P = 39500 \frac{5}{\sqrt{25-1}} \quad \text{(equation 1, Chapter XVIII)}$$
$$= 40030 \text{ lbs.}$$

Assume $\qquad q = 800$ and $n = 1.3$;

then

$$d = \sqrt{\frac{40300}{1.3 \times 800}} = 6\tfrac{1}{4} \text{ ins.};$$

and

$$l = 1.3d = 8\tfrac{1}{4} \text{ ins. (nearly)};$$

checking for stress, we have

$$s = 1.27 \frac{40300 \times 8.25}{(6.25)^3} = 1740;$$

which is within safe limits.

Stuffing Boxes. Where a rod having either a rotary or reciprocating motion passes through the wall of a vessel containing a fluid under pressure, provision must be made to prevent leakage. The device used for this purpose is called a stuffing box. Thus the piston rod and valve rod of a steam engine are examples of reciprocating parts, while shafts of steam turbines and centrifugal pumps are examples of rotating parts requiring stuffing boxes. The packing used is generally some soft fibrous material, but for high pressures and temperatures the so-called metallic packings are preferred.

In Fig. 19–17 is shown a common form of stuffing box for rods from $1\tfrac{1}{2}$ ins. diameter and up. It consists of the stuffing box or casing A and the gland B. The gland compresses the packing in box and thus tightens it about rod. The proportions may be as follows: if d is diameter of rod, then internal diameter of box is

$$D = 1.25d + \tfrac{1}{2} \text{ in.}$$

The length of box is made from

$$h = d + 1 \text{ in. to } 1.5d + 1 \text{ in.},$$

high pressures requiring the longer boxes. For the smaller sizes of rods, say up to 3 or 4 ins. the oval gland with two bolts is used, while large stuffing boxes commonly have glands with circular flanges requiring 3 or 4 bolts. The diameter of bolts may be

$$\delta = \frac{\sqrt{d}}{4} + \tfrac{3}{8} \text{ in.}$$

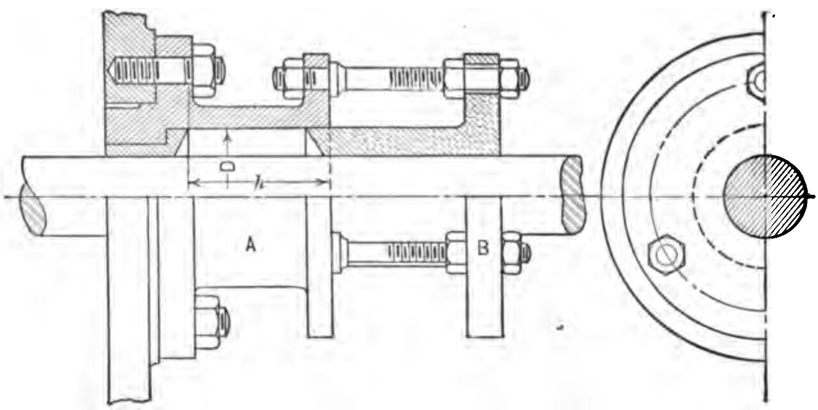


FIG. 19–17.

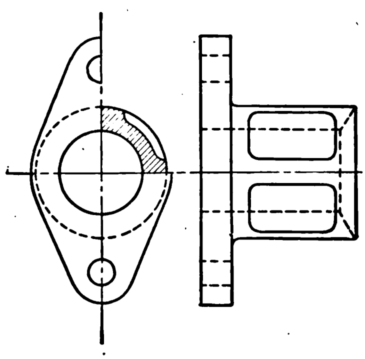


FIG. 19–17*a*.

For rods less than 2 ins. diameter the screwed stuffing box shown in Fig. 19–18 is used. The gland is forced into box by means of a cap threaded over box. The proportions may be about the same as those given above.

Large rods generally have the gland lined with a brass bushing and if there is considerable side pressure on rod, as, for instance,

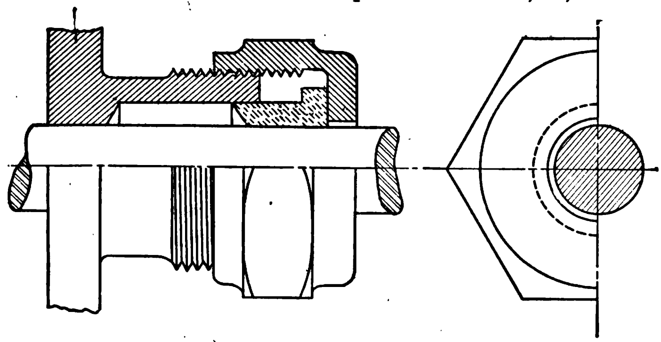


FIG. 19–18.

due to weight of a heavy piston, the bottom of casing also is bushed, as shown in Fig. 19–17.

A type of metallic stuffing box is shown in Fig. 19–19. The packing consists of two sets of split rings having a wedge-shaped

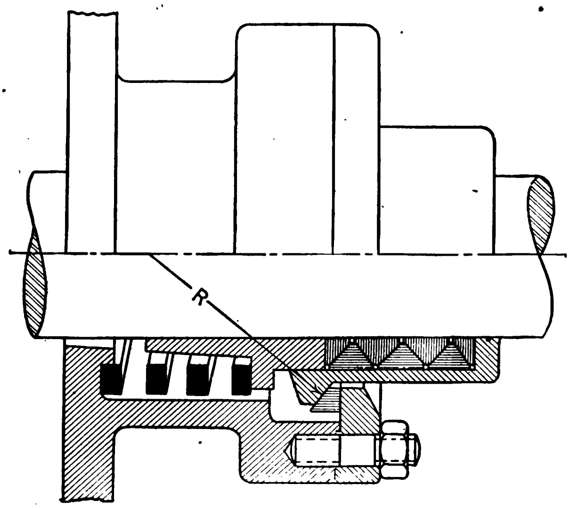


FIG. 19–19.

section. A spring acting on a sleeve presses the rings together, thus tightening the inner set around rod. It will be noticed that the sleeve carrying the rings is on a spherical seat; this enables

the rings to adjust themselves to the piston rod even if cylinder and rod axes are not in alignment.

PROBLEMS

1. The cross-head shoes of a steam engine are 6½ by 10 ins. The engine has a cylinder 12 ins. in diameter by 18-in. stroke. The length of connecting rod is 40½ ins. Determine the maximum bearing pressure per square inch on cross-head, if the steam pressure is 150 lbs. per square inch.

2. Determine the dimensions of the wrist-pin in problem 1. Assume bearing pressure is 800 lbs. per square inch and $n = 1.3$. Check design for bending stress.

3. The piston rod of engine in problem 1 is 2 ins. in diameter. Make a sketch design of stuffing box for same, to scale.

4. Draw to scale a sketch design of piston for engine of problem 1, similar to Fig. 19–4.

5. In a gas engine the maximum pressure in cylinder is 375 lbs. per square inch. The cylinder diameter is 10¾ ins. The piston-pin is 3½ ins. in diameter by 5½ ins. long. Determine bearing pressure and check-pin for stress.

CHAPTER XX

HOISTING MACHINERY DETAILS

Chains. Link chains for hoisting machines are generally forged to gauge, the material used being wrought or mild steel. In Fig. 20–1 are shown good proportions for hoisting chains. The short pitch link is better as the chain is more flexible and when wound on drum or sheaves is subjected to smaller bending stresses.

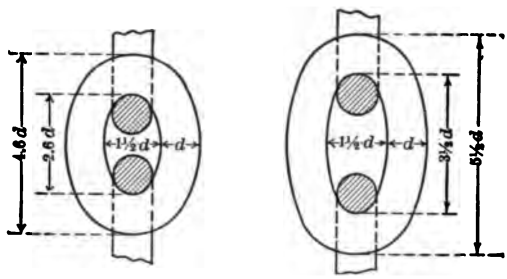


FIG. 20–1.

The long pitch chain is lighter and cheaper for the same strength.

The allowable tensile stress in chain links may be 5000 lbs. per square inch for power-driven hoists to 8000 lbs. for hand-operated machines. This gives approximately

$$P = 12{,}000d^2 \text{ (hand driven)},$$
$$P = 8000d^2 \text{ (power driven)},$$

where d is diameter of bar from which chain is forged and P is safe load.

Sheaves for Chains. In Fig. 20–2 are shown various forms of sheaves for guiding chains. The pitch diameter should not be less than $20d$. The sheave shown at a is one most commonly used. The high flanges at the sides are necessary only where

chain is subjected to much vibration or where it is led off at an angle to the plane of the sheave.

Chain Drums. These are provided usually with a helical groove to guide the chain as in Fig. 20–3. A clearance of $\frac{1}{8}$ to $\frac{3}{16}$ in. is allowed between the links of adjacent turns. The minimum diameter of drum is the same as that for sheaves. The minimum thickness is generally determined by the possibility of obtaining a sound casting and is not less

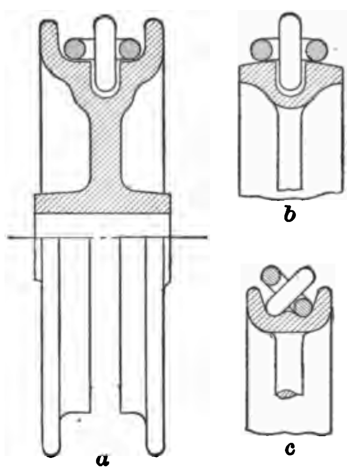


FIG. 20–2.

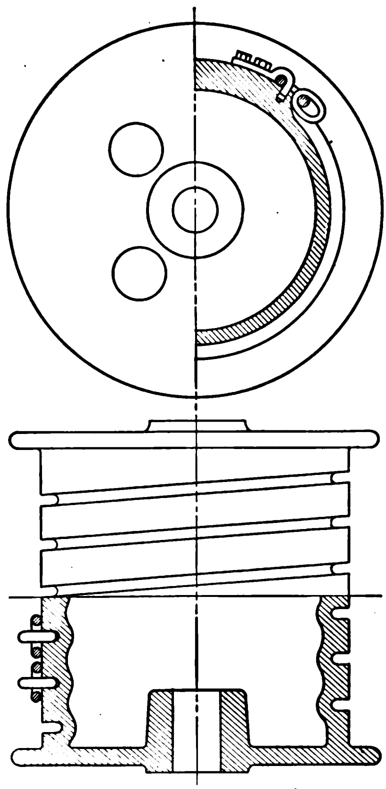


FIG. 20–3.

than $\frac{1}{2}$ in. The following empirical equation may be used for obtaining the thickness t in terms of drum diameter D.

$$t = .02D + \tfrac{3}{8} \text{ in.} \qquad (1)$$

The drum may be checked for maximum stress. The greatest bending moment occurs when load acts at middle of drum and is

$$M = \frac{PL}{4};$$

the twisting moment is

$$T=\frac{PD}{2};$$

then the equivalent bending moment is

$$M_e=\tfrac{1}{2}\left\{\frac{PL}{4}+\sqrt{\left(\frac{PL}{4}\right)^2+\left(\frac{PD}{2}\right)^2}\right\}; \quad . \quad . \quad (2)$$

and the stress is

$$s=\frac{M_e}{z}=\frac{M_e}{\frac{\pi}{32}\left(\frac{D^4-D_1{}^4}{D}\right)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

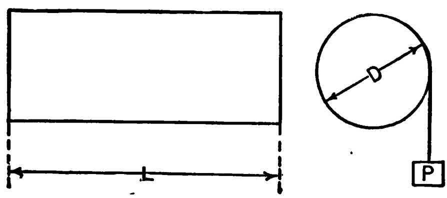


FIG. 20-4.

where D_1 is the internal diameter of drum. The length L of drum is determined by the maximum distance load is to be hoisted. When the load is in its lowest position two or three turns of chain should still remain on drum.

Chain Wheels. With heavy loads and high lifts chain drums are very cumbersome on account of their great weight and large dimensions. In this case chain wheels are used, as their width is independent of the height of lift and their diameter may be much smaller than that of drums. Such a wheel is shown in Fig 20-5. If l is pitch of chain and N is number of teeth on wheel, then the pitch diameter of wheel is

$$D=\sqrt{\left(\frac{l}{\sin\frac{90^\circ}{N}}\right)^2+\left(\frac{d}{\cos\frac{90^\circ}{N}}\right)^2}. \quad . \quad . \quad . \quad . \quad (4)$$

For wheels having eight or more teeth the second term becomes negligible and we may write

$$D=\frac{l}{\sin\frac{90^\circ}{N}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hooks. There are numerous forms of hooks used as experience developed them for the particular use to which they are to be put.

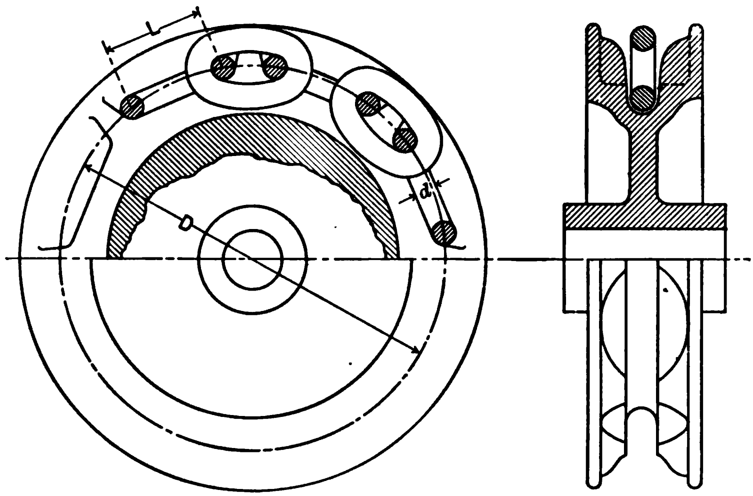


FIG. 20-5.

In Fig. 20-6 and 20-7 are shown the most common types. The material used is high-grade wrought iron or mild steel. The

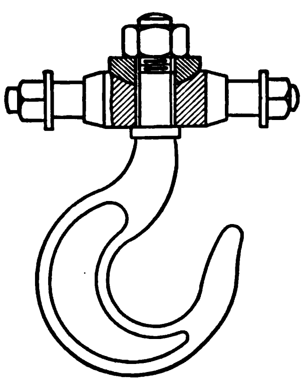


FIG. 20-6.

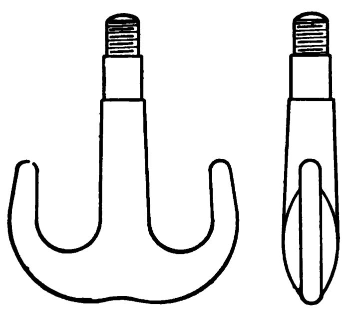


FIG. 20-7.

following proportions may be used, referring to Fig. 20-8. If d_1 be the outside diameter of threaded portion of hook, then, allow-

ing a stress of 8000 lbs. per square inch and keeping in mind that the root diameter is about $.83d_1$, we have

$$P=\frac{\pi}{4}\times(.83d_1)^2\times 8000;$$

or approximately

$$d_1=\frac{\sqrt{P}}{65}; \qquad (6)$$

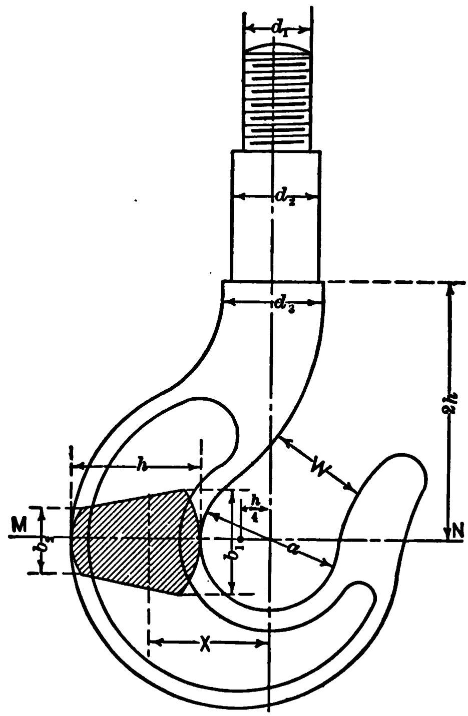


FIG. 20-8.

then the other dimensions expressed in terms of d_1 may be $d_2=\frac{5}{4}d_1$, $d_3=\frac{3}{2}d_1$, $b_1=d_1$, $b_2=\frac{1}{2}b_1$, $h=2d_1$, $W=1.6d_1$ and $a=2\frac{1}{4}d_1$. The maximum stress occurs in the section MN. This section is subjected to a direct tension, s_t, due to the load and a bending stress, s_b, due to the moment arm of the load about the center of gravity of the section. Then

$$s_t=\frac{P}{A};$$

where A is the area of the section and

$$s_b = \frac{M}{z} = \frac{Px}{z};$$

where x is the distance from center of book to neutral axis of section MN and the total stress is

$$s = s_t + s_b = P\left(\frac{1}{A} + \frac{x}{z}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For the section shown in Fig. 20–8

$$z = .192 b_2 h^2.$$

The proportions shown above will give a stress of about 18,000 lbs. per square inch at the section MN. This is allowable if first-class material be used.

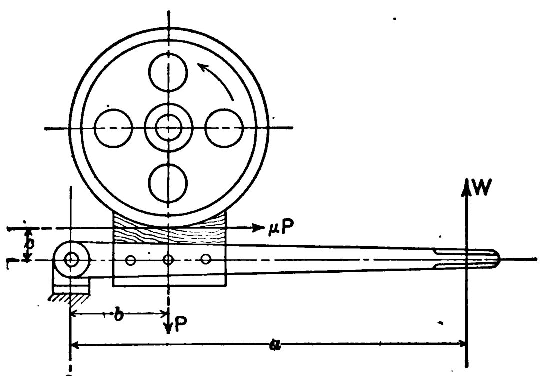


FIG. 20–9.

Brakes. The simplest form of brake is the *block brake*, as in Fig. 20–9. The friction of the wood block against the brake drum prevents its rotation.

Let W = weight on lever,
P_t = force to be braked at rim of drum,
P = pressure of block against drum,
μ = coefficient of friction.

Since, in order to have equilibrium, the moment of all forces about the fulcrum must be equal to zero, we have

$$Wa - Pb - \mu Pc = 0,$$

when direction of rotation is as indicated, if in opposite direction

$$Wa-Pb+\mu Pc=0;$$

but the friction at rim of drum must at least be equal to P_t or

$$\mu P=P_t;$$

then

$$P=\frac{P_t}{\mu};$$

and therefore

$$W=\frac{P_t}{a\mu}(b\pm\mu c). \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This equation shows that the weight W necessary to hold load in equilibrium is proportional to P_t the tangential force at rim of brake drum. It is therefore desirable to place brake on a high-speed shaft if possible, since that gives the smallest value of P_t. If c is made zero the last term in equation (8) disappears. Thus when brake drum rotates in either direction by making c equal to zero, the weight W required will be the same for both directions of rotation. The coefficient of friction may be taken at .4 for wood or leather on cast iron, if surfaces are dry, and .2 for iron on iron. To avoid one-sided pressure on the brake drum two blocks may be arranged on diametrically opposite sides of drum, as in Fig. 20–10. By the use of grooved friction surfaces the braking effect may be greatly increased as the coefficient of friction μ in equation (8) is replaced by

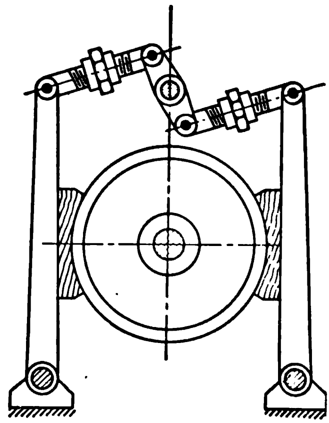


FIG. 20–10.

$$\mu'=\frac{\mu}{\sin\alpha+\mu\cos\alpha}.$$

Band Brakes. In this type of brake the drum is encircled by a metallic band which sometimes is lined with wood or other friction surfaces. This band may be tightened around drum by

means of levers. Fig. 20–11 shows the simplest form of band brake. One end of band is attached to fulcrum of lever while the other end is fastened to a pin a short distance out on lever. It is evident that the friction of band on drum is identical to that

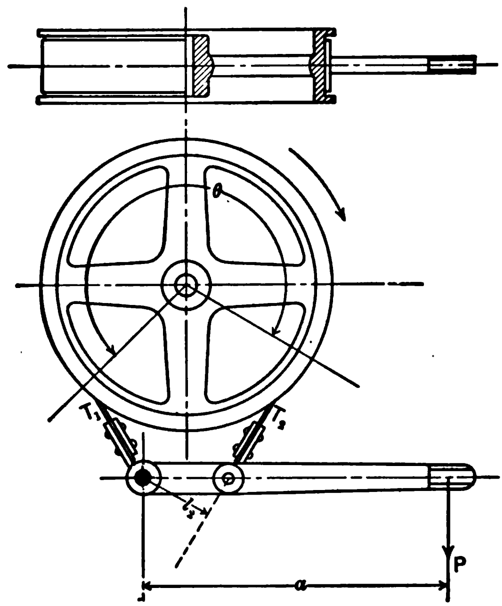


Fig. 20–11.

of a belt on pulley and mathematically it may be treated in the same way.

Let T_1 = tension on tight side of band,
T_2 = tension on slack side of band,
l_2 = lever arm of tension T_2,
θ = angle subtended by band in radian measure,
μ = coefficient of friction,
e = base of Napierian logarithms (2.718),
P = force or load at end of lever,
P_t = force to be braked at brake drum;

then (see page Appendix B)

$$\frac{T_1}{T_2} = e^{\mu\theta};$$

$$P_t = T_1 - T_2 = T_2(e^{\mu\theta} - 1);$$

and taking moments about fulcrum of lever

$$Pa - T_2 l_2 = 0;$$

$$\therefore \quad P = \frac{T_2 l_2}{a} = \frac{P_t}{e^{\mu\theta} - 1} \frac{l_2}{a}. \quad \ldots \ldots \quad (9)$$

In this equation P is the load which will just hold a force P_t acting at the rim of brake drum, in equilibrium. If the direction of rotation be reversed P will be larger since now the greater tension T_1 has a lever arm l_2 about the fulcrum. Proceeding as above, we obtain for this case the equation

$$P = \frac{P_t e^{\mu\theta}}{e^{\mu\theta} - 1} \frac{l_2}{a}. \quad \ldots \ldots \ldots \quad (10)$$

The value of μ may be taken at .18 for a steel band. This gives the following values of $e^{\mu\theta}$:

PORTION OF CIRCUMFERENCE ENCLOSED BY BAND

	.1	.2	.3	.4	.5	.6	.7	.8	.9
$e^{\theta\mu} =$	1.12	1.25	1.40	1.57	1.76	1.97	2.21	2.47	2.77

The brake band is designed for tension with an allowable stress of 8000 to 10,000 lbs. per square inch. The thickness should not exceed a maximum of $\frac{3}{16}$ in. while the breadth should not be greater than 3 or 4 ins., as it is difficult to make a wide band lie in contact along its entire width. The diameter of drum is usually from 8 to 16 ins. unless brake is part of hoisting drum.

The differential band brake is illustrated in Fig. 20–12. Here both ends of band are attached to pins located at different distances from the fulcrum. A counterweight is shown to balance the weight of the lever. Taking moments about fulcrum, we have

$$Pa + T_1 b_1 - T_2 b_2 = 0;$$

and therefore

$$P = \frac{T_2 b_2 - T_1 b_1}{a}; \quad \ldots \ldots \ldots \quad (11)$$

substituting for T_1 and T_2 we obtain

$$P = \frac{P_t}{a} \left(\frac{b_2 - b_1 e^{\mu\theta}}{e^{\mu\theta} - 1} \right) \quad \ldots \ldots \ldots \quad (12)$$

EXAMPLE. In Fig. 20–13 is shown in diagrammatic form a hoist to lift a load of 5000 lbs. Find the force P required to hold the load in equilibrium.

The force P_t which must be applied at rim of brake drum is

$$P_t = \frac{4 \times 8}{12 \times 7} \times 5000 = 1900 \text{ lbs.}$$

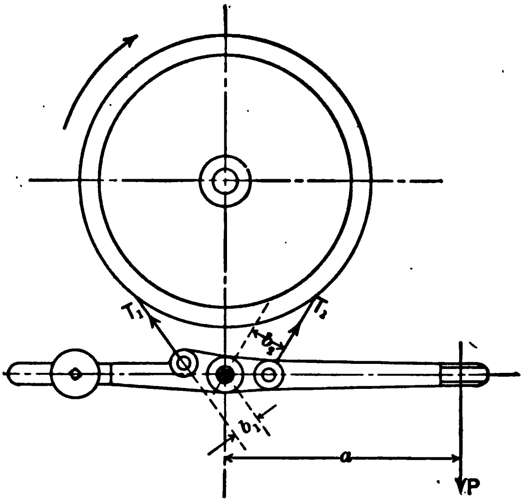


FIG. 20–12.

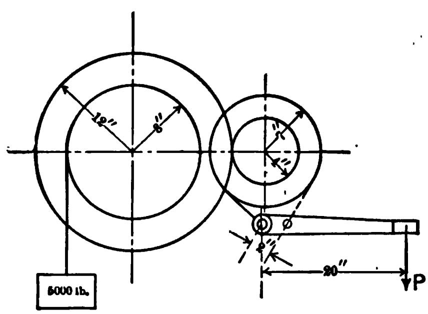


FIG. 20–13.

Assume that the band subtends .7 of the drum circumference, then

$$\frac{T_1}{T_2} = e^{\mu\theta} = 2.2 \quad \text{(from table)};$$

and from equation (9)

$$P=\frac{1900}{2.21-1}\times\frac{2}{20}=170 \text{ lbs.}$$

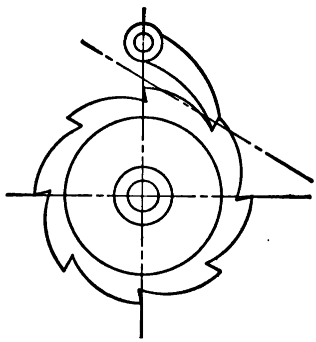


FIG. 20-14.

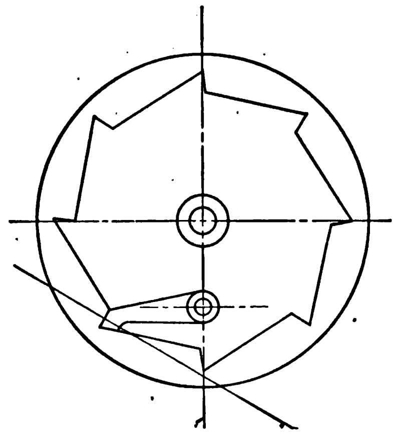


FIG. 20-15.

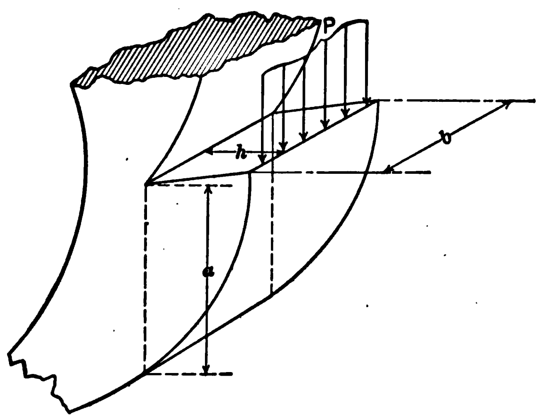


FIG. 20-16.

The maxium tension in band is

$$T_1=\frac{P_t}{1-\frac{1}{e^{\mu\theta}}}=\frac{1900}{.647}=2940 \text{ lbs.}$$

Ratchets and pawls are used in hoisting machinery to prevent a load from descending when the operating power is cut off. The ratchet wheel may have external teeth as in Fig. 20–14 or internal as in Fig. 20–15. With external teeth the angle of the tooth face should be such that when the pawl is in engagement the common normal will pass between the axes of pawl and ratchet; for internal teeth the normal must lie outside these axes as shown in figures. The teeth are calculated for bending. Assuming that

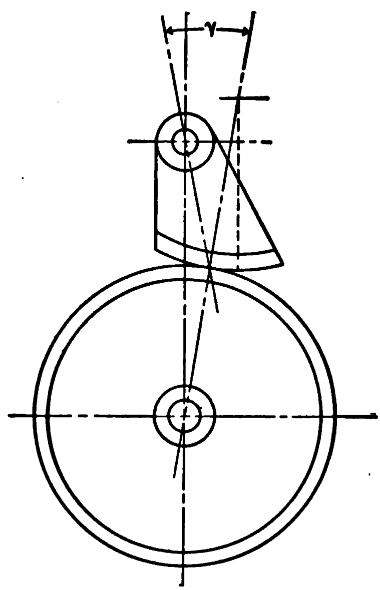


FIG. 20–17.

the force P acts at point of tooth, if b is breadth of wheel, then, from Fig. 20–16 the bending moment is

$$M = Ph = s\tfrac{1}{6}ba^2;$$

and

$$s = \frac{6Ph}{ba^2}. \qquad \qquad (13)$$

The friction stop, Fig. 20–17, has the advantage of noiseless operation. The angle γ must be such that tan γ is less than the coefficient of friction μ of the two surfaces in contact. For cast iron on cast iron $\mu = .15$, thus tan $\gamma < .15$ and therefore

$\gamma < 8°$. To avoid this small angle the wheel may be made grooved as in Fig. 20–18. In this case γ is determined by the equation

$$\tan \gamma < \frac{\mu}{\sin \alpha + \mu \cos \alpha}.$$

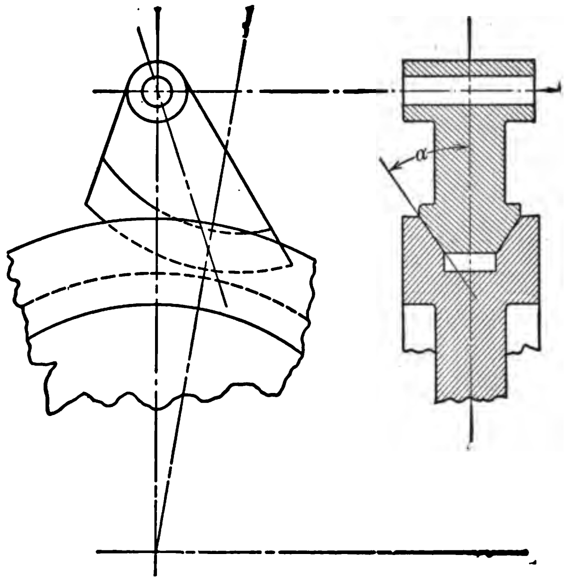


FIG. 20–18.

PROBLEMS

1. The drum of a hoist is 20 ins. in diameter by 44 ins. long. The thickness of metal is $\frac{3}{4}$ in. Determine maximum stress in drum when lifting 30,000 lbs.

2. Make a sketch design of a hook according to proportions given in Fig. 20–8. The hook to carry a load of 10 tons. Determine the stress in section *MN*.

3. In a power-driven hoist the drum shaft makes 1 revolution to 5 of the driving shaft. The hoisting drum is 18 ins. in diameter, the brake drum which is on driving shaft is 15 ins. in diameter. The load is 12,000 lbs. Design a block brake similar to Fig. 20–9, assuming $\mu = .25$. Design to be such that one man can operate brake for either direction of rotation. Make sketch of your design.

4. In a band brake similar to Fig. 20–11, the diameter of brake drum is 30 ins., $a = 36$ ins., $l_2 = 4$ ins., $\theta = \frac{3}{4}$ of circumference and $\mu = .22$. Determine the load which can be braked at the brake drum if force at end of lever is 60 lbs.

CHAPTER XXI

SPRINGS

UNLIKE other machine parts springs are designed to give large deflections under the action of external forces. They are used, first: to maintain definite relative positions between two machine members until the forces acting to change this relative position exceed a desired limit, as in valves, governors, etc.; second, for absorbing energy due to sudden applications of force as in automobiles, railway cars, etc.; third, to store energy as in clock springs, and fourth, to measure forces as in spring balances.

Springs may be divided into two general classes: those in which the chief stress is a bending stress and those in which the chief stress is torsional. In the following discussion let

P = safe load,
f = deflection due to load P,
E = modulus of elasticity,
E_s = modulus of torsional elasticity.

Flat Spring. (Fig. 21–1.) This is the simplest form of spring. It is considered to be a cantilever loaded at one end, then

$$P=\frac{bh^2}{6}\frac{s}{l}; \quad \dots\dots\dots\dots \quad (1)$$

and

$$f=\frac{2}{3}\frac{l^2s}{hE}. \quad \dots\dots\dots\dots \quad (2)$$

Another form of this spring is of triangular shape. (Fig. 21–2.) The safe load is the same as above. The deflection is

$$f=\frac{l^2s}{hE}. \quad \dots\dots\dots\dots \quad (3)$$

Laminated or Leaf Springs. These consist of a number of layers held together by a clip as shown in Fig. 21-3. They may

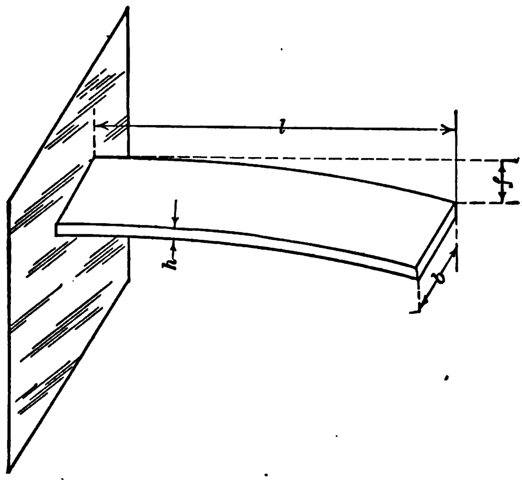


FIG. 21-1.

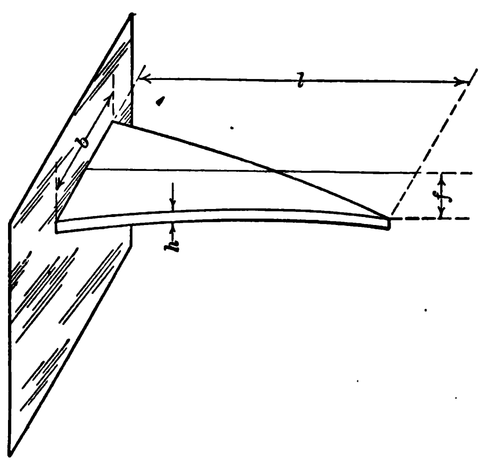


FIG. 21-2.

be considered equivalent to the lozenge-shaped plate. Due to the friction between leaves the deflection will be somewhat less than calculated. Such springs are largely used in motor and

railway cars, where they are generally of the form shown in Fig. 21–4. If n is the number of leaves then

$$P=\frac{2}{3}\frac{sbh^2}{l}n; \qquad (4)$$

and

$$f=\frac{3}{8}\frac{Pl^3}{bh^3nE}. \qquad (5)$$

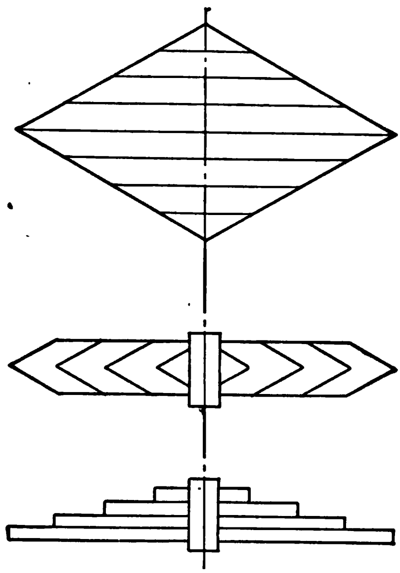


FIG. 21–3.

Spiral Springs. (Fig. 21–5.) These springs usually have a rectangular section. The safe load is

$$P=\frac{bh^2s}{6r}. \qquad (6)$$

The deflection is

$$f=r\omega=\frac{2rls}{hE}. \qquad (7)$$

Helical or Coil Springs. (Fig. 21–6.) In this type of spring the chief stress is torsion. For circular section of wire the safe load is

$$P=\frac{\pi d^3s_s}{8D}; \qquad (8)$$

and the deflection of *one* coil under a load P is

$$f=\frac{8D^3P}{E_s d^4}. \qquad \qquad (9)$$

In the design of a coil spring the maximum load P and the total deflection f' required is known; it then remains to find the diameter D of the coil, the diameter d of the wire, and the number of coils n. As we have only two equations connecting

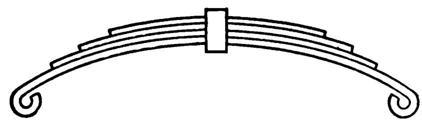


FIG. 21-4.

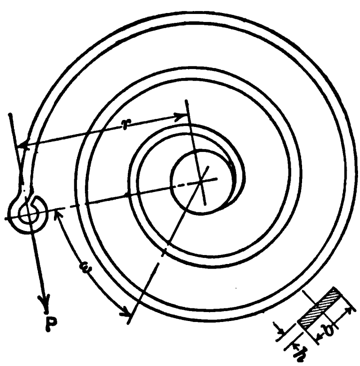


FIG. 21-5.

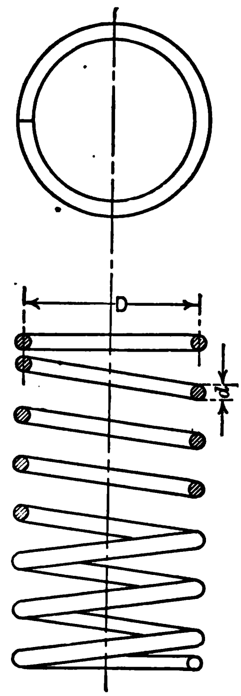


FIG. 21-6.

the last three of these quantities it is evident that we may assume any one of the three and calculate the other two. It is thus possible to have a great variety of springs to satisfy the same conditions as to load and deflection. To facilitate getting practical results the following tables have been calculated. Let the ratio $\frac{D}{d}=k$, then equation (8) becomes

$$P=\frac{\pi}{8}\frac{d^2}{k}s_s.$$

In Table 1 values of P have been calculated for values of k from 4 to 20. This will include all customary sizes of springs. It has been assumed that $s_s = 100{,}000$, a value which may be used for modern high-grade alloy spring steels. For any other value of s_s it is simply necessary to multiply the tabular value by the desired s_s divided by 100,000.

The deflection has been tabulated in the same way in Table 2. If in equation (9) we substitute its value from equation (8) and then substitute k for $\frac{D}{d}$ we obtain:

$$f = \frac{\pi d s_s k^2}{E_s}.$$

For $s_s = 100{,}000$ and $E_s = 12{,}500{,}000$ this equation becomes $f = .0251k^2d$. The deflections given in table are based on these values.

For rectangular sections of wire (Fig. 21-7) the safe load is

$$P = \frac{4}{9}\frac{b^2 h s_s}{D}; \qquad (10)$$

and the deflection per coil is

$$f = 4\pi D^2 \frac{b+h^2}{bh^2}\frac{s_s}{E_s}.$$

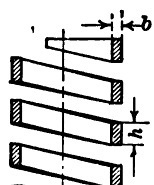


FIG. 21-7.

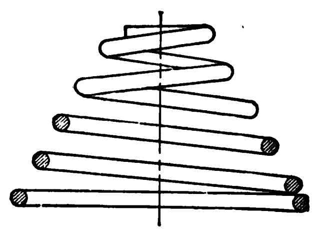


FIG. 21-8.

Volute or Conical Springs. (Fig. 21-8.) These are springs in which the diameters of the successive coils are uniformly reduced. Since the maximum stress is in the largest coil the safe loads are the same as those of coil spring of the same section and equal diameter.

Material of Springs. High-carbon steel is the material most commonly employed in springs. This is a grade containing from .8 to 1.2 per cent of carbon. In the smaller diameters of wire

SAFE LOAD ON SPRINGS

B. & S. Gauge.	Diam., Ins.	k											
		4	5	6	7	8	9	10	12	14	16	18	20
24	.02010	3.96	3.17	2.64	2.26	1.88	1.75	1.58	1.32	1.13	.94	.87	.79
23	.02257	4.92	3.93	3.28	2.83	2.46	2.18	1.96	1.64	1.42	1.23	1.09	.98
22	.02535	6.69	5.35	4.46	3.82	3.35	2.97	2.67	2.23	1.91	1.67	1.48	1.33
21	.02846	7.95	6.35	5.30	4.55	3.98	3.53	3.17	2.65	2.28	1.99	1.76	1.58
20	.03196	10.0	8.00	6.67	5.72	5.00	4.44	4.00	3.33	2.86	2.50	2.22	2.00
19	.03589	12.6	10.1	8.40	7.20	6.30	5.60	5.05	4.20	3.60	3.15	2.80	2.53
18	.04030	15.9	12.7	10.6	9.10	7.95	7.06	6.03	5.30	4.55	3.97	3.53	3.02
17	.04526	19.6	15.65	13.1	11.2	9.80	8.72	7.82	6.55	5.60	4.90	4.36	3.28
16	.05082	25.3	20.2	16.9	14.4	12.6	11.2	10.1	8.45	7.20	6.30	5.60	5.05
15	.05707	31.8	25.4	21.2	18.2	15.9	14.2	12.7	10.6	9.10	7.95	7.10	6.35
14	.06408	40.4	32.3	26.9	23.1	20.2	17.9	16.1	13.5	11.5	10.1	8.95	8.05
13	.07196	49.9	39.9	33.3	28.5	24.9	22.2	19.9	16.6	14.3	12.5	11.1	9.95
12	.08081	63.8	51.0	42.5	36.5	31.9	28.3	26.5	21.3	18.2	15.9	14.2	13.2
11	.09074	80.5	64.4	53.7	46.0	40.2	35.7	32.2	26.8	23.0	20.1	17.8	16.1
10	.10189	101.4	81.1	67.6	58.0	50.7	45.0	40.6	33.8	29.0	25.3	22.5	20.3
	$\frac{1}{8}$	153	122	102	87.5	76.5	67.3	61.0	51.0	44	38.2	33.7	30.5
	$\frac{3}{16}$	345	276	230	197	173	153	138	115	98	86.5	76.5	69.0
	$\frac{1}{4}$	612	490	408	350	306	272	245	204	175	153	136	122
	$\frac{5}{16}$	962	770	642	550	482	429	385	321	275	241	214	192
	$\frac{3}{8}$	1460	1144	921	835	730	613	572	460	418	365	306	286
	$\frac{7}{16}$	1880	1500	1252	1075	940	835	750	626	537	470	417	375
	$\frac{1}{2}$	2450	1960	1638	1400	1225	1043	880	819	700	612	521	440
	$\frac{5}{8}$	3830	3060	2560	2208	1915	1704	1530	1280	1104	957	852	765
	$\frac{3}{4}$	5500	4400	3670	3140	2750	2450	2200	1835	1570	1375	1225	1100

DEFLECTION OF SPRINGS

B. & S. Gauge.	Diam. Ins.	k											
		4	5	6	7	8	9	10	12	14	16	18	20
24	.02010	.0081	.0126	.0181	.0247	.0323	.0408	.0504	.0727	.0990	.128	.163	.202
23	.02257	.0091	.0142	.0220	.0278	.0363	.0458	.0567	.0815	.111	.145	.186	.227
22	.02535	.0102	.0159	.0229	.0312	.0407	.0515	.0636	.0917	.125	.165	.206	.255
21	.02846	.0114	.0179	.0258	.0351	.0458	.0579	.0715	.103	.140	.183	.232	.286
20	.03196	.0128	.0200	.0288	.0393	.0513	.0650	.0803	.115	.157	.205	.250	.321
19	.03589	.0144	.0225	.0324	.0441	.0577	.0730	.0902	.130	.177	.231	.292	.360
18	.04030	.0162	.0253	.0365	.0496	.0648	.0820	.101	.146	.198	.259	.328	.405
17	.04526	.0182	.0284	.0409	.0557	.0727	.0920	.113	.167	.223	.291	.368	.455
16	.05082	.0204	.0318	.0458	.0625	.0815	.103	.127	.183	.250	.326	.413	.510
15	.05707	.0230	.0359	.0517	.0705	.0920	.116	.144	.203	.282	.368	.466	.575
14	.06408	.0260	.0406	.0585	.0797	.104	.132	.163	.234	.319	.417	.526	.651
13	.07196	.0288	.0450	.0684	.0882	.115	.146	.180	.259	.352	.461	.583	.720
12	.08081	.0324	.0506	.0728	.0993	.130	.164	.202	.292	.396	.518	.656	.810
11	.09074	.0364	.0569	.0819	.111	.146	.184	.227	.327	.446	.582	.738	.910
10	.10189	.0408	.0637	.0918	.125	.163	.206	.255	.367	.500	.654	.826	1.02
	$\frac{1}{8}$	.0502	.0784	.113	.154	.201	.252	.314	.451	.615	.803	1.03	1.25
	$\frac{3}{16}$	.0755	.1170	.169	.231	.310	.381	.470	.680	.925	1.20	1.53	1.88
	$\frac{1}{4}$	.1004	.157	.226	.307	.402	.507	.628	.905	1.23	1.61	2.03	2.51
	$\frac{5}{16}$	.1256	.196	.282	.384	.502	.635	.785	1.13	1.54	2.01	2.54	3.14
	$\frac{3}{8}$	.1505	.236	.340	.462	.605	.765	.945	1.36	1.85	2.42	3.06	3.78
	$\frac{7}{16}$	.1760	.274	.396	.527	.704	.892	1.10	1.58	2.15	2.82	3.56	4.40
	$\frac{1}{2}$	.2070	.314	.452	.615	.803	1.03	1.25	1.81	2.46	3.21	4.06	5.02
	$\frac{5}{8}$	.2514	.392	.565	.770	1.000	1.27	1.57	2.26	3.08	4.02	5.08	6.29
	$\frac{3}{4}$	.3180	.471	.678	.924	1.20	1.52	1.88	2.71	3.69	4.82	6.10	7.53

or rod it will have an ultimate tensile strength of about 150,000 lbs. per square inch, after proper heat treatment; its elastic limit being then around 100,000 lbs. per square inch. The demands of the motor-car industry have developed a number of special alloy spring steels which have tensile strengths ranging from 200,000 to 300,000 lbs. per square inch and correspondingly high elastic limits. Brass and bronze is also used extensively for springs, especially in the small sizes. Besides these metals wood and rubber is sometimes used for the same purpose.

PROBLEMS

1. Design a helical spring for a safety valve so that it will open when pressure under valve is 150 lbs. per square inch. The valve is 3 ins. in diameter. The spring is to be about 5 ins. long when compressed.

2. Design the helical spring for the exhaust valve of a 50 H.P. gas engine. The following conditions to be observed: Force on spring when valve is closed 200 lbs., lift of valve $1\frac{1}{4}$ ins. Number of coils 15. What is the load of spring when valve is open?

✓ 3. A flat spring 6 ins. long, 1 in. wide and $\frac{1}{16}$ in. thick has a load of 12 lbs. acting at its end. Determine deflection of spring. ($E = 30{,}000{,}000$.)

CHAPTER XXII

MATERIALS OF MACHINERY

THE design of machinery includes a great deal more than the knowledge of calculating the various component parts, a thorough acquaintance with the materials used in their construction is quite essential as well as the shop methods used in giving the parts their required shape. In the following pages a brief outline of the most commonly used materials is given.

Iron. By far the most important material which enters into the construction of machines is iron. It is used in any of three forms, viz.: cast iron, wrought iron and steel. The raw material for all these is iron ore. The most important ores are magnetite or black oxide of iron, which when pure contains 72.4 per cent of iron. The famous "Swedish iron" is made from this ore. Haemetite, or red oxide of iron, which is the chief source of supply in this country, contains about 68 per cent of iron. Siderite, or ferrous carbonate, which is an important source of supply for the English iron industry, contains about 48 per cent of iron.

The smelting process takes place in a high furnace called blast furnace. The charge consists of the ore, fuel and a flux. The fuel may be coke, charcoal, or coal which is free from sulphur, the flux being limestone. The proportions of each depend upon the ore used. The materials are charged at the top of the furnace, which is 50 to 100 ft. high, and maintained at a constant height by adding new material every ten to twenty minutes as the melting proceeds. Near the base of the furnace a blast of air is admitted, which in the usual hot blast method has been preheated to a temperature varying from 500 to 1200° F. The pressure of the blast ranges from $3\frac{1}{2}$ to 10 lbs. per square inch. When the hearth is filled with the molten metal the blast is shut off and the furnace is tapped.

The product of the blast furnace is known as pig iron. This is classed as grey, mottled, or white, according to the appearance

of the fracture. Grey pig iron has a granular appearance, a dark grey color and is soft, easily machined and filed. It is especially useful for foundry purposes, being classified as Nos. 1, 2, 3, etc., according to the greyness. Mottled iron has the appearance of a matrix of white with grey spots, white cast iron has a white and often crystalline appearance. It is extremely hard and brittle, melts more readily, but flows less freely than grey iron, giving off sparks in abundance. It is less suitable for making castings, but is used largely in the production of wrought iron.

Pig iron contains considerable quantities of other elements besides iron; it is these impurities which largely determine the value of the iron for different purposes. The most important of these substances are carbon, silicon, manganese, sulphur and phosphorus. The carbon exists in two forms, as combined carbon and as graphitic carbon. The total carbon in pig iron is from 2 to 5 per cent. The greater the proportion of combined carbon the harder and whiter the resulting product. Graphitic carbon makes the iron grey and soft and raises the melting point. Sulphur is looked upon as the chief enemy of the iron and steel maker. Its presence in wrought iron or steel causes red shortness, that is, the metal cannot be worked well above a red heat, but cracks under the hammer.

Cast Iron. This is the product of the blast furnace after it has been remelted in the cupola of the foundry and is formed into the desired shape by pouring into a mold. A good deal of experience and judgment is required by the foundryman in mixing the various grades of pig iron to give the resulting product the desired qualities. Thus a steam engine cylinder must be of a close-grained grey cast iron soft enough to be easily machined but not so soft as to wear readily due to the rubbing of the piston. An ornamental casting for architectural purposes must be made of a pig iron containing more phosphorus as this makes it expand slightly on solidifying and thus gives a casting true to the mold.

No metal used by the machine constructor varies so much in strength and soundness as cast iron. This is particularly true of its resistance to tension, which varies from about 14,000 to about 30,000 lbs. per square inch. Its elastic limit is low, ordinarily about one-third of its ultimate resistance. For these reasons a high factor of safety should be employed, for dead tensile

loads the stress allowed may be taken at 4000 to 5000 lbs. per square inch. The compressive strength of cast iron is much higher, ranging generally from 80,000 to 100,000 lbs. per square inch.

Malleable Cast Iron. Malleable castings are produced by partially decarbonizing cast iron. This is done by placing the castings in cast-iron boxes and covering them with an oxidizing agent such as haemetite ore. They are heated and maintained at a red heat, for a number of days, the air being entirely excluded. The cast iron used for this purpose is high in combined carbon, viz.: white cast iron; a portion of this carbon is converted by this process into graphite carbon. In this country the so-called "black heart" process is used in which the casting has a tough skin resembling wrought iron in its properties and a soft dark-grey interior. Such castings are used where something better than cast iron is wanted; these castings approach the strength of steel, but are more easily machined. Malleable iron should carry a central load of 3000 lbs. in a transverse test of a 1-in. square bar on supports 12 ins. apart with a deflection of $\frac{1}{2}$ in.

Chilled Castings. The greater the amount of combined carbon in cast iron the harder the resulting casting. Sudden cooling tends to prevent the formation of free or graphitic carbon. This fact is made use of in the production of chilled castings. In this process the mold or a portion of it is of metal with a thin coating of loam; the metal is rapidly cooled by this metal wall and the resulting casting has a very hard surface, the effect of the chill extending a distance of $\frac{1}{8}$ to $\frac{1}{2}$ in. below the surface. Such castings are used where great hardness is needed to resist wear and a strong tough interior to withstand shock. Thus car wheels and the rolls used in rolling mills are often chilled castings.

Wrought Iron. This is the purest form of commercial iron containing in the best grades up to 99.6 per cent iron and only 0.4 per cent of impurities, chiefly carbon and slag. Wrought iron is made by oxidizing the impurities of pig iron in a reverberatory furnace. This method is called the puddling process. The raw pig iron used must be of a special grade, low in sulphur, phosphorus, and silicon; charcoal pig is the material which best fulfills these conditions.

In the reverberatory or puddling furnace the fuel, unlike the

blast furnace, does not come in contact with the charge of pig iron; the flames and hot gases of combustion heating the charge to the melting point. The hearth on which the charge, 400 or 500 lbs., is placed is lined with an oxidizing material, called fetling, such as iron oxide ore; in about half an hour it is partially melted, forming a pasty mass which becomes less and less fluid as the carbon and other impurities are removed. It is stirred with an iron tool called a rabble so as to bring all parts under the influence of the fetling. The metal is then collected in balls or blooms weighing about 80 lbs.

It is now in a soft spongy condition quite unfit for use. These blooms are subjected to a hammering or squeezing process which removes a portion of the slag present. The resulting product is known as puddled bar and is still of too coarse a structure to be used. The puddled bar is cut up into short lengths which are piled into a faggot, reheated to a welding heat and again hammered and rolled into bars, which are commercially known as merchant bar. This process of cutting, piling, reheating and rolling is repeated a number of times, depending on the quality of the wrought iron desired. Each repetition makes the metal purer and more uniform in structure, giving it a fibrous appearance. The name of the resulting product gives the number of times the above process has been repeated; thus best bar is made from merchant bar, best best from best bar and treble best from best best.

Although steel has largely displaced wrought iron, partly because it is about 20 per cent cheaper, there are many engineers who prefer wrought iron for specific purposes, as for rivets, bolts, etc. Good wrought iron should have a tensile strength of 45,000 to 55,000 lbs. and an elongation of 25 per cent before rupture occurs, and have an elastic limit of about one-half its tensile strength, and it should bend double, cold, without cracking.

Case Hardening. A process by means of which the surface of a wrought-iron article is made to absorb carbon and is thereby given the property of becoming hard upon heating and suddenly quenching, is called case hardening. The method is as follows: The articles to be case hardened are packed in cast- or wrought-iron boxes together with some powdered material rich in carbon such as bone dust or animal charcoal; often these are mixed with some cyanide. Each article must be entirely surrounded and in

close contact with the carbonizer. The box is closed by a cover which is sealed down air tight with clay. The boxes are now placed on a furnace and heated to a good orange heat. This temperature is maintained from two to twenty-four hours, depending on the size of the articles and depth to which they are to be case hardened. The boxes are now withdrawn from the furnace and allowed to cool slowly. The articles are then removed and cleaned. They are then heated in a muffle furnace to a cherry red and quenched in oil or water.

Steel. The carbon contents of iron largely determine the difference between cast iron, wrought iron and steel. Thus wrought iron is very low in carbon contents, from .10 to .40 per cent; cast iron is rich in carbon, containing from 3 to 5 per cent of it. Steel is intermediate between these, containing from .15 to 1.8 per cent carbon. According to the amount of carbon it is classified into low carbon or mild steel and high carbon or tool steel.

Mild Steel. The raw material for mild steel is pig iron. There are two methods of manufacture, the Bessemer process and the open-hearth process. Mild steel can be easily forged, rolled and welded, but cannot be hardened except by the case hardening process.

The **Bessemer** process of making steel is carried on in a pear-shaped vessel called the Bessemer converter. The bottom of this vessel is perforated with many small holes through which a powerful blast of air enters. The converter is partly filled with a bath of molten pig iron. The action of the air blast is to burn out nearly all the silicon manganese and carbon. The converter is lined with either an acid or a basic lining and is mounted on trunnions so that it can be tilted to any angle. After the impurities have been burned out it is necessary to recarburize the bath. This is usually done by adding some material rich in carbon, after the converter has been emptied into a ladle. The time required for a heat is from eight to fifteen minutes in the acid process and from twenty to twenty-five minutes in the basic process, the capacity of the converters is from 8 to 25 tons per heat.

Open-hearth Steel. In this process the charge consists of either pig iron or of pig iron and scrap and ore. This is melted in the hearth of a regenerative Siemens furnace. Here, as in the

Bessemer process there is a basic and an acid process according to the lining being acid, silica bricks and sand, or basic, magnesite brick covered with crushed dolomite. These furnaces are made in sizes from 5 tons up to 200 or over. A heat takes from five to ten hours. The production of open-hearth steel is rapidly increasing and it seems to be supplanting Bessemer steel in many fields. This reason for this is that it is a sounder and more homogeneous metal.

High-carbon Steel. This steel is manufactured either by the crucible process or by the cementation process; the raw material being either wrought iron or mild steel. The cementation process is used chiefly in the Sheffield district in England. In this process bars of pure wrought iron are impregnated with carbon. For this purpose bars $2\frac{1}{4}$ to 3 ins. wide, $\frac{5}{8}$ to $\frac{3}{4}$ in. thick and about 12 ft. long are packed in layers separated by charcoal in a cementing furnace. The furnace, which is sealed air-tight and heated externally, attains its full temperature in three or four days and is maintained at this for about seven to twelve days, depending on the carbon content desired in the steel. The charge is then cooled, this requiring from four to six days. The resultant product is known as blister steel. It is of a coarse crystalline structure. The blister steel is broken into short lengths, piled into faggots, these are reheated and hammered or rolled into bars, this product being known as shear steel. This greatly improves the quality of the steel, and the process may be repeated several times, depending on quality of product desired.

In the crucible process high-carbon steel is produced by melting the charge, consisting usually of Swedish wrought iron, in crucibles with the addition of some carbon. The crucibles, which have a capacity of about 100 lbs., are heated in a gas furnace of the regenerative type. It requires $3\frac{1}{2}$ to 4 hours to convert a charge into steel. High-carbon steel produced by the crucible process is known as cast steel. This as well as the high-carbon steel produced by the cementation process is used for all varieties of cutting tools and wherever great hardness and good wearing qualities are required. The high-carbon steels possess the property of hardening when heated and quenched.

Alloy Steels. Alloy steels are those which owe their properties chiefly to the presence of an element, or elements other than carbon.

Nickel Steel. This is the most widely used of all alloy steels. The nickel contents vary from 1.5 to 4.5 per cent, the most usual proportion being 3 to 3.75 per cent, the carbon varying from .2 to .5 per cent. Nickel steel is largely used for engine forgings (connecting rods, etc.), locomotive axles, shafting and motor car frames and engine parts. Nickel steel has a higher elastic limit than carbon steel; this enables it to resist repeated and alternating stresses better. The ultimate tensile strength is increased by about 4000 lbs. per square inch above that of carbon steel by each per cent of nickel added up to 5 per cent. It is somewhat harder than ordinary steel but not so much but that it can be readily machined. It is also used for making steel castings, as it is comparatively free from blow holes and melts at a lower temperature than carbon steel. Nickel steel has a very small coefficient of expansion. When the amount of nickel is 36 per cent the coefficient is less than that of any other metal or alloy known, being practically zero. This alloy has been patented and goes under the trade name of Invar. Platinite is another nickel steel alloy containing 42 per cent of nickel and having the same coefficient of expansion as glass.

Chrome Steel. This alloy when in the hardened state combines the properties of great hardness with high elastic limit. The chromium ranges from 1 to 2 per cent with carbon from .8 to 2 per cent. It is used for making armor-piercing projectiles, for very hard plates such as are used in burglar-proof safes, and where for reasons of wear great hardness is required, as in automobile gears.

Vanadium Steel. The manufacture of this the newest of the alloyed steels is still in its infancy. An addition of only .1 to .15 per cent of vanadium greatly increases the strength of steel. Its chief characteristic is its resistance to torsional and vibratory stresses. It is therefore useful where lightness and strength are required, as in springs and in auto axles, etc.

Tungsten Steel. This steel is also known as self-hardening, air-hardening or high-speed steel. The amount of tungsten used is as high as 24 per cent but usually ranges from 5 to 10 per cent with carbon from .4 to 2 per cent. These steels cannot be annealed by any known process, and will maintain a cutting edge even at a red heat. Messrs. F. W. Taylor and M. White made a very

extensive set of tests with this steel for the Bethlehem Steel Works. The results of these tests were the beginning of a new era in machine-shop practice.

Non-ferrous Metals. Next to iron the most important metal that is used by the machine constructor is copper. Its great value lies in the ease with which it alloys with other metals to form an almost infinite number of very useful alloys. Just as different bodies are soluble to varying degrees in water or even entirely insoluble, so metals possess the property of alloying with others in different degrees, some perform this readily in all proportions, as for instance copper with tin, copper with zinc, iron with manganese, and gold with silver; others only in a limited degree, as iron with zinc, or iron with copper; and others not at all, as iron with lead or with silver.

The purpose of alloying metals is to produce a material possessing certain valuable properties not possessed by the individual constituents. In general the tenacity is increased if certain ratios of the constituents are not exceeded. The melting-point is usually lower than the mean of the elements which make up the alloy. Thus tin melts at 230° C., and lead at 330° C., while the alloy tin 63 parts and lead 37 parts melts at 181° C. Heat and electric conductivity are always decreased.

Brass. Nominally brass should consist of two metals, copper and zinc only, but many varieties contain small amounts of iron, tin, arsenic and lead. As copper and zinc will alloy in almost all proportions a great variety of brass alloys exist. The most useful may be divided into three grades of soft, medium, and hard brass having approximately the following composition, soft, copper 80 and zinc 20; medium, copper 70 and zinc 30; and hard, copper 60 and zinc 40. The medium is used for sheets, tubing and wire. It is readily worked cold but not so easily hot. The hard is well adapted for casting, as it gives sharp and clean castings and is cheap on account of the high zinc contents. It forges well in the hot state and is therefore adapted for drop forgings. The 80 and 20 alloy is used where a special soft material is desired.

Brass castings are soft whether suddenly or slowly cooled. Rolling or otherwise mechanically working brass makes it hard and brittle. It is therefore necessary to anneal it, if further reduction is required. There are a number of brass alloys which have

been patented or are known by special trade names. Thus Muntz metal consists of copper 56 to 63 per cent and zinc 44 to 37 per cent. Delta metal, 55 copper, 43½ zinc, 1 iron and small quantities of lead and phosphorus.

Bronze. This alloy, which has been in the service of man for unknown centuries, is primarily a combination of copper and tin. There is an exceedingly wide range in which these two metals alloy and a corresponding variation in physical properties of the resultant product. Standard commercial bronze alloys contain tin and copper in ratios varying from 1 tin and 4 copper, to 1 tin and 12 copper. Thus gun-metal bronze contains copper 89 to 92 per cent and tin 11 to 8 per cent; bearing-bronze copper 82 to 88 per cent and tin 18 to 12 per cent.

Phosphor bronze contains from ½ to 1 per cent of phorphorus. This alloy is much used in high-grade construction for bearings, also for springs, gear blanks, etc.

Tobin bronze is used most frequently in the rolled shape for bolts, pump piston rods, etc., where it is subject to corrosion by salt water. As used in the U. S. navy its composition is copper 59 per cent, zinc 38.4 per cent, tin 2.16 per cent, lead .31 per cent, and iron .11 per cent.

Manganese bronze is an alloy of copper and zinc improved by small quantities of manganese, iron, phosphorus and tin. It varies considerably in composition and may be either soft and ductile or hard and strong. It is almost universally used for the propellers of large steamboats, as it is strong, resists corrosion by salt water, and has a very smooth surface when cast. Its tensile resistance is from 65,000 to 75,000 lbs. per square inch. There are numerous other bronzes each possessing some desirable property, as for example chromax bronze has a tensile strength equal to that of high-grade steel, viz.: 70,000 to 80,000 lbs. per square inch. It is also an alloy of copper, zinc, nickel, aluminum and chromium.

White-metal Alloys. These metals are the so-called anti-friction metals used for lining bearings. There is an exceedingly large variety on the market, some of high-sounding name and inferior quality. They may be divided into four classes: 1st, tin, antimony, and copper alloys; 2d, lead and antimony alloys; 3d, lead, tin and antimony alloys; and 4th, zinc, tin and anti-

mony alloys. There are, however, numerous variations possible; copper sometimes displaces the zinc or antimony in the last two classes.

The best known and most widely used of these white-metal alloys are the Babbitt metals. The original Babbitt metal was an alloy containing about 89 per cent tin, 7 to 8 per cent antimony and 3 to 4 per cent copper; but very little of the so-called genuine Babbitt metal that is sold is made according to the original formula. As a matter of fact almost any white metal that is used for lining bearings is called Babbitt metal. Tin being by far the most expensive of the metals used in these bearing alloys, the much cheaper lead and antimony compounds are preferred.

These white-metal alloys can be easily melted in an ordinary ladle. It is therefore a simple process to pour the alloy around the journal into the bearing body, and for small or medium-sized bearings this is usually done. For low or medium speeds and bear ing pressures that are not too high, depending on the hardness of the alloy, these white-metal alloys make the best possible bearing linings.

Aluminum, one of the newest metals at the command of the engineer, is of particular advantage where great lightness is desirable, as its weight is only about one-third tha+ of iron. It is very malleable and ductile; it can be cast in molds similar to cast iron. In the form of castings it has a tensile resistance of 15,000 lbs. per square inch which can be increased to 25,000 by rolling or hammering. Combined with copper it forms some valuable alloys known as aluminum bronzes, the tensile strength of which run up to 90,000 lbs per square inch.

Timber. The term timber is applied both to standing trees and to lumber of large dimensions. Lumber is a general term applied to wood which has been cut into any of the various sizes used in construction work. The trees from which our lumber is derived may be divided into broad-leaf or hard wood and needle leaf or soft wood.

If a section of a tree be examined it will be seen to consist of a somewhat darker interior core called heart wood, a lighter exterior portion called sap wood surrounding this bark. The heart wood and sap wood are arranged in concentric rings, one of which

is formed each year. Dense narrow rings which indicate slow growth produce strong wood.

Green wood contains a good deal of moisture, the effect of which is materially to reduce the strength. It is therefore necessary to dry the wood before using. If done in the open it is said to be air dried, a method which may take from several months to two or three years. If it is dried in an enclosed chamber to which heat is applied it is kiln-dried lumber, this being a much more rapid method. The process of drying wood is called seasoning.

The strength of timber depends on the amount of heart wood or sap wood and such defects as knots, checks, cracks, and any break in the continuity of fibers. Wood is much stronger both in tension and compression along the grain than across the grain.

A brief description of the physical properties and uses of the more common woods is given below.

White Ash. Heavy, hard, strong, elastic, becoming brittle with age. Not durable in contact with soil. Uses: Agricultural implements, handles, oars, interior and cheap cabinet work. Red and green ash are frequently used as substitutes for the more valuable white ash.

White Cedar. Soft, light, fine grained, durable in contact with soil, but not very strong or tough. Uses: cooperage, boat building, shingles, posts, etc.

Cypress. Light, hard, close grained but brittle. An exceedingly durable wood, easily worked, takes a high polish with satiny appearance. Uses: interior finish and cabinet work, construction, etc.

White Elm. Heavy, hard, strong and tough. Not easily shaped and warps in drying. Takes good polish. Uses: car, wagon and ship building, bridge timbers, sills, furniture, and barrel staves.

Hickory. Heaviest, hardest and toughest of American woods. Very flexible. Uses: handles, and best wood implements.

Hard Maple. Tough, heavy, hard and strong. Takes good polish. Not durable when exposed. Uses: furniture, flooring, pegs, interior finish, etc.

Soft Maple. Light, brittle, and easily worked, moderately strong. Takes high polish. Not durable when exposed. Uses: woodenware, turned work, flooring, interior finish, etc.

White Oak. Heavy, strong, tough and close grained. Takes a high polish. Uses: framing structures, ship building, interior finish and furniture making.

White Pine. Light, soft, and straight grained. Easily worked, but not very strong. Uses: interior finish, pattern making, etc.

Yellow Pine (long-leaf). Heavy, strong, hard, tough, durable if dry and well ventilated, but not in contact with ground. Uses: heavy framing timbers, flooring, etc. Short-leaf yellow pine is used as a substitute for the long leaf.

Oregon Pine (Douglas Fir). Hard, strong but of variable quality, durable but difficult to work. Used for all kinds of construction.

Black Spruce. Light, soft, close and straight grained. Used for piles, framing timbers, submerged cribs, etc.

APPENDIX A

If in Fig. 7–10 an elementary ring of width dr be taken then the moment of the friction on this ring is the total pressure on the surface multiplied by the coefficient of friction and by the radius r_1 of the ring, or

$$dM_f = \mu p \pi r_1{}^2 dr;$$

$$\therefore \quad M_f = \int_0^r \mu p \pi r_1{}^2 dr \quad \ldots \ldots \ldots \quad (1)$$

$$= \frac{2\pi \mu p r^3}{3}; \quad \ldots \ldots \ldots \quad (2)$$

but

$$\pi r^2 p = P_a;$$

or

$$p = \frac{P_a}{\pi r^2};$$

substituting this value in equation (2) we obtain

$$M_f = \tfrac{2}{3}\mu P_a r.$$

If the discs are in the form of a ring of outside radius R and inside radius r, then equation (1) above becomes

$$M_f = \int_r^R \mu p \pi r_1{}^2 dr = \frac{2\mu p \pi}{3}(R^3 - r^3); \quad \ldots \quad (3)$$

but $P_a = \pi(R^2 - r^2)$ and substituting this value (3) reduces approximately to

$$M_f = P_a \mu \frac{R+r}{2}. \quad \ldots \ldots \ldots \quad (4)$$

APPENDIX B

Let Fig. 9–2 represent an elementary length of belt on the rim of its pulley. Relatively to the pulley it is in equilibrium under the forces shown, that is, a tension T at one end, $T+dT$ at the other end, and friction dF, or

$$T+dT=T+dF. \quad \ldots \ldots \ldots \quad (1)$$

If dN is the normal pressure between belt and pulley, then

$$dF=\mu dN;$$

but from the parallelogram of forces

$$dN=(2T+dT)\sin\frac{d\theta}{2};$$

and since $d\theta$ is an infinitesimal angle we can substitute for the sine the angle itself, then

$$dN=(2T+dT)\frac{d\theta}{2}.$$

Neglecting the product of the infinitesimals,

$$dN=Td\theta;$$

therefore

$$dF=\mu Td\theta;$$

then from equation (1)

$$dT=dF;$$

$$\therefore \quad dT=\mu Td\theta;$$

integrating we obtain

$$\int_{T_2}^{T_1}\frac{dT}{T}=\int_0^{\theta}\mu d\theta;$$

or

$$\log_e T_1-\log_e T_2=\mu\theta;$$

$$\therefore \quad \frac{T_1}{T_2}=e^{\mu\theta}.$$

APPENDIX C

In Fig. 14–5 is shown a portion of the pipe. The total radial pressure on an arc subtending the angle $d\theta$ is

$$MN = rd\theta Lp;$$

and the component of this pressure normal to plane AB is

$$\begin{aligned} ON &= MN \cos \theta \\ &= rLp \cos \theta d\theta. \end{aligned}$$

The total vertical component R is therefore

$$\begin{aligned} R &= \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} rLp \cos \theta d\theta \\ &= 2rLp = DLp. \end{aligned}$$

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